24P107 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P PHY1 C02 - MATHEMATICAL PHYSICS - I

(Physics)

(2019 Admissions)

Time : 3 Hours Maximum : 30 Weightage

Section A

Answer ***all*** questions. Each question carries 1 weightage.

1. What are the characteristics of orthogonal curvilinear coordinates?
2. Express Laplacian operator in cylindrical coordinates.
3. Define Hermitian matrix and Unitary matrix.
4. Show that eigen values of a Hermitian matrix are real and eigen vectors belonging to different eigen values are orthogonal.
5. Define Levi-civita three index symbol.
6. What is the significance of the Gram Schmidt orthogonalisation procedure?
7. State Abel's Theorem.
8. Define Laplace transformation. Find the laplace transformation of $f(t) = e^{ikt}$ where k is a constant.

(8 × 1 = 8 Weightage)

Section B

Answer any ***two*** questions. Each question carries 5 weightage.

9. Derive the expression for gradient,divergence and curl in general curvilinear co-ordinate system.Use the result to find the expressions for the same in circular cylindrical and spherical polar co-ordinates.
10. Given an ODE of the form y" + P(x)y' + Q(x)y = 0 . Let one of the two independent solutions be y_1. Explain how you can find the second solution.
11. Define Bessel Function, Derive the Orthogonality condition of Bessel Function and show that $\int_0^1 x\,[J_n(\alpha x)]^2 dx = \frac{1}{2}[J_n{}'\alpha]^2$
12. (a) Show that $P_n(x) = \frac{1}{2^n\,.\,n!}\,\frac{d^n}{dx^n}(x^2-1)^n$

    (b) Prove That $P_n(x)$ is the coefficient of z^n in the expansion of $(1-2xz+z^2)^{-\frac{1}{2}}$ in the ascending powers of z

(2 × 5 = 10 Weightage)

Section C

Answer any ***four*** questions. Each question carries 3 weightage.

13. Transform the unit vectors i,j,k into their components in cylindrical coordinate system.

14. Check the singularity of Bessel equation $x^2y'' + xy' + (x^2 - m^2)y = 0$ at $x=0$ and $x=\infty$

15. Show that $\Gamma(1/2) = \sqrt{\pi}$

16. Prove That $\int_0^1 \frac{dx}{\sqrt{1-x^4}} = \frac{\sqrt{\pi}}{4} \frac{\Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{3}{4}\right)}$

17. Show that (1) $H_n(-x) = (-1)^n H_n(x)$ (2) $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$ (3) $H_{2n+1}(0) = 0$

18. Express the function $f(x) = \begin{cases} 1 & ,when\ |x| \leq 1 \\ 0 & ,when\ |x| > 1 \end{cases}$. Hence evaluate $\int_0^\infty \frac{\sin\lambda\cos\lambda x}{\lambda} d\lambda$

19. State and explain convolution theorem on Fourier transform.

(4 × 3 = 12 Weightage)

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