23P361 (Pages: 2) Name……………………..

Reg. No…………………..

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

(CUCSS-PG)

(Regular/Supplementary/Improvement)

CC19P MST3 C12/ CC22P MST3 C10 – STOCHASTIC PROCESSES

(Statistics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

PART A

Answer any ***four*** questions. Each question carries 2 weightage.

1. Write down the Chapmann-Kolmogorov equations for Markov Chains
2. Explain discrete time Branching processes
3. Define Poisson process. What is the interarrival time distribution of Poisson process?
4. What are non-homogenous Poisson process and compound Poisson process?
5. Explain Renewal reward process
6. What is the M/G/I queuing system?
7. Describe Brownian motion process

(4 × 2 = 8 Weightage)

PART B

Answer any ***four*** questions. Each question carries 3 weightage.

8. Consider a Poisson process with rate λ, show that inter arrival time between two successive events is Exponentially distributed with mean 1/λ .
9. Consider a counting process { $N(t)$}. If $N(t) = n,$ then show that n arrival times $S_1, S_2, \dots, S_n$ have the same distribution as the order statistics corresponding to n independent and identically distributed random variables uniformly distributed on the interval (0,t).
10. Let X_n be the time between (n-1)th and nth arrival with distribution $F_n(t),$ of a counting process $\{N(t)\}$. Then derive the renewal function is $m(t) = \sum_{n=0}^{\infty} F_n(t)$.
11. Let $N(t)$ be a counting process and X_n be the time between (n-1)th and nth arrival with distribution $F_n(t)$ and mean μ. Then with probability one, show that $\frac{N(t)}{t} \to \frac{1}{\mu}$, as $t \to \infty$
12. Obtain the steady state probability distribution P_n, limiting probability that there are n customers in the system, for a Single-Server exponential queuing system having finite capacity.

13. Derive the expected queue size and expected waiting time in a M/M/1 model.

14. For a continuous time Markov process with transition probability function $P_{ij}(t)$, show that $P_{ij}(t+s) = \sum_{k=0}^{\infty} P_{ik}(t) P_{kj}(s)$

(4 × 3 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

15. State and derive the limiting probabilities for a birth and death process

16. If {N(t)} is a Poisson process derive auto-correlation between $N(t)$ and $N(t+s)$, t, $s>0$.

17. State and prove elementary renewal theorem.

18. Derive the first hitting time distribution of a Brownian motion Process.

(2 × 5 = 10 Weightage)

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