23P301 (Pages: 2) Name: ……………………..

Reg. No: …………………..

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P MTH3 C11 – MULTIVARIABLE CALCULUS AND GEOMETRY

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Suppose X is a vector space, $\dim X = n$ and X has a basis. Show that every basis of X consists of n vectors.
2. Suppose f maps an open set $E \subset R^n$ into R^m and f is differentiable at a point $x \in E$. Show that the partial derivatives $(D_j f_i)(x)$ exists for $1 \leq j \leq n, 1 \leq i \leq m$.
3. Prove that a linear operator A on R^n is invertible if $\det[A] \neq 0$.
4. Determine whether the curve $\gamma(t) = (e^{kt}\cos t, e^{kt}\sin t)$ is regular or not.
5. Show that any open disc in the xy-plane is a surface.
6. Let $f: S_1 \to S_2$ be a diffeomorphism. If σ_1 is an allowable surface patch on S_1, then show that $f \circ \sigma_1$ is an allowable surface patch on S_2.
7. Show that the area of a surface patch is unchanged by reparametrization.
8. Define the normal curvature and geodesic curvature of a unit-speed curve γ on an oriented surface S.

(8 × 1 = 8 Weightage)

PART B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT I

9. Prove that to every $A \in L(R^n, R^1)$ corresponds a unique $y \in R^n$ such that $Ax = x.y$.
10. State and prove the contraction principle.
11. If $[A]$ and $[B]$ are $n \times n$ matrices, then show that $\det([A][B]) = \det[A]\det[B]$.

UNIT II

12. Show that a parametrized curve has a unit-speed reparametrization if and only if it is regular.

13. Let γ be a unit-speed curve in R^3 with constant curvature and zero torsion. Prove that γ is a parametrization of (part of) a circle.

14. Show that the Möbius band is not orientable.

UNIT III

15. Show that the level surface $\{(x, y, z) \in R^3 \mid x^2 + y^2 + z^4 = 1\}$ is a smooth surface.

16. Compute the second fundamental form of $\sigma(u, v) = (u, v, u^2 + v^2)$.

17. State and prove the Meusnier's theorem.

(6 × 2 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

18. State and prove the implicit function theorem.

19. i) Show that any regular plane curve with constant curvature is part of a circle.

    ii) Compute the curvature κ, principal normal vector $\boldsymbol{n}$, binormal vector $\boldsymbol{b}$ and torsion τ of the curve $\gamma(t) = \left(\frac{4}{5}\cos t,\ 1 - \sin t, -\frac{3}{5}\cos t\right)$.

20. i) Find the tangent plane of the surface patch $\sigma(u, v) = (u, v, u^2 - v^2)$ at $(1, 1, 0)$.

    ii) Show that the transition maps of a smooth surface are smooth.

21. i) Show that every point on the unit sphere S^2 is umbilic.

    ii) Let S be a (connected) surface of which every point is an umbilic. Show that S is an open subset of a plane or a sphere.

(2 × 5 = 10 Weightage)

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