23P304 (Pages: 2) Name: ……………………..

Reg. No: …………………..

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P MTH3 C14 – PDE AND INTEGRAL EQUATIONS

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Solve the equation $u_x + u_y = 2$ subject to the initial condition $u(x,0) = x^2$.
2. Show that the Cauchy problem $u_x + u_y = 1,\ u(x,x) = x$ has infinitely many solutions.
3. If $u(x,y)$ is a harmonic function in a domain D, then show that $u \in C^\infty(D)$.
4. State the necessary condition for the existence of a solution to the Neumann problem.
5. Show that $\frac{d^n}{dx^n} I_n(x) = (n-1)!\, f(x)$, where $I_n(x) = \int_a^x (x-\xi)^{n-1} f(\xi) d\xi$.
6. Using Green's function, transform the differential equation $\frac{d^2y}{dx^2} + xy = 1,\ \ y(0) = y(1) = 0$ to an integral equation.
7. Make use of an appropriate formula to show that the iterative procedure will converge if $|\lambda| < 3$ for the integral equation $y(x) = \lambda \int_0^1 x\,\xi\ y(\xi) d\xi + 1$.
8. Consider the problem $u_{tt} - u_{xx} = 0,\ -\infty < x < \infty,\ t > 0$

$$u(x,0) = f(x) = \begin{cases} 0, & -\infty < x < -1 \\ x+1, & -1 \le x \le 0 \\ 1-x, & 0 \le x \le 1 \\ 0, & 1 < x < \infty \end{cases}$$

$$u_t(x,0) = g(x) = \begin{cases} 0, & -\infty < x < -1 \\ 1, & -1 \le x \le 1 \\ 0, & 1 < x < \infty \end{cases}$$

Evaluate u at the point $(1, \frac{1}{2})$. Also discuss the smoothness of the solution $u(x,y)$.

(8 × 1 = 8 Weightage)

PART B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT-1

9. Solve the eikonal equation ${u_x}^2 + {u_y}^2 = {n_0}^2$, ($n_0$ is a constant) with the initial condition $u(x,2x) = 1$.

10. Reduce to canonical form $u_{xx} - 2\sin x\, u_{xy} - cos^2x\, u_{yy} - cosx\, u_y = 0$ and find the general solution.

11. Show that for a fixed $T > 0$, the Cauchy problem for the one-dimensional homogeneous wave equation in the domain $-\infty < x < \infty, 0 \le t \le T$ is well posed for $f \in C^2(\mathbb{R})$, $g \in C^1(\mathbb{R})$.

UNIT-2

12. Solve $u_t - u_{xx} = 0,\ 0 < x < \pi,\ t > 0$ subject to $u(0,t) = u(\pi,t) = 0\,, t \ge 0$ and

$$u(x,0) = \begin{cases} x, & 0 \le x \le \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \le x \le \pi \,. \end{cases}$$

13. State and prove the weak maximum principle.

14. Let $u \in C_H$ be a solution of the heat equation $u_t = k\Delta u,\ t > 0$ in Q_T. Prove that u achieves its maximum on $\partial_P Q_T$.

UNIT-3

15. Write a short note on Neumann series.

16. Find the characteristic values and corresponding characteristic functions for the equation $y(x) = \lambda \int_0^{2\pi} \sin(x+\xi)\, y(\xi)\, d\,\xi$.

17. Show that $y'' + A(x)\, y' + B(x)y = f(x)$ with the initial conditions $y(a) = y_0,\ y'(a) = {y_0}'$ can be transformed in to a Volterra equation of second kind.

(6 × 2 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Using the method characteristics, solve $-yu_x + xu_y = u$ subject to the initial condition $u(x,0) = \psi(x)$.

19. Solve the problem $u_{tt} - u_{xx} = t^7$ subject to $u(x,0) = 2x + sinx,\ u_t(x,0) = 0$, $-\infty < x < \infty,\ t > 0$.

20. Solve $u_{tt} - 4u_{xx} = 0,\ 0 < x < 1,\ t > 0$ subject to $u_x(0,t) = u_x(1,t) = 0\,, t \ge 0$ and $u(x,0) = cos^2\pi x,\ u_t(x,0) = sin^2\pi x\, cos\pi x,\ 0 \le x \le 1$.

21. Solve the integral equation $y(x) = 1 + \lambda \int_0^1 x\xi\, y(\xi) d\xi$ by the iterative method.

(2 × 5 = 10 Weightage)

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