23U316S (Pages: 2) Name……………………

Reg. No……………………

THIRD SEMESTER B.Voc. DEGREE EXAMINATION, NOVEMBER 2024

(Information Technology)

CC18U GEC3 ST08 – PROBABILITY DISTRIBUTIONS

(2018 to 2021 Admissions - Supplementary)

Time: Three Hours Maximum: 80 Marks

Section A

Answer ***all*** question. Each question carries 1 mark.

Fill up the blanks:

1. The characteristic function of a random variable is ……………….
2. If X is a random variable then V(5X + 4) = ………………
3. If X and Y are independent random variables then E(XY) = ……………..
4. The p.d.f of geometric distribution is ……………….
5. If A and B are independent events, then f(x,y) = …………………

Write true or false:

6. Moment generating function exists for all the random variables.
7. For a Poisson distribution mean and variance are equal.
8. The mean, median and mode of a normal distribution are identical.
9. Cov(X,Y) = 0,implies that X and Y are independent.
10. Central limit theorem can only be applied for Poisson distribution.

(10 × 1 = 10 Marks)

Section B

Answer any ***eight*** question. Each question carries 2 marks.

11. Define mathematical expectation.
12. Write the mean and variance of a binomial distribution with parameters (20,0.4)
13. Identify the distribution which possesses lack of memory property.
14. What is the mean of geometric distribution having the pdf $f(x) = 2^{-x}, x = 1,2,3, \dots \dots$?
15. Define standard normal distribution.
16. Give any two situations where Poisson distribution is used.
17. Show that $V(X) = E(X^2) - \{E(X)\}^2$.
18. Suppose X and Y are two random variables. Describe joint density function.
19. Define Cauchy distribution.
20. What are the assumptions of Central Limit Theorem?

21. Derive the moment generating function of Exponential distribution.

22. What do you mean by convergence in probability?

(8 × 2 = 16 Marks)

Section C

Answer any ***six*** questions. Each question carries 4 marks.

23. Derive the relationship between raw and central moments.

24. Define conditional variance.

25. Define bivariate distribution function. What are its properties?

26. Show that $V(X_1+X_2) = V(X_1-X_2)$ if X_1 and X_2 are independent random variables.

27. Prove that Cauchy-Schwartz inequality.

28. State and prove addition theorem on expectation if X and Y are two random variables.

29. Suppose $X \sim N(\mu_1, \sigma_1^2), Y \sim N(\mu_2, \sigma_2^2)$. If X and Y are independent random variables then $X + Y \sim N(\mu_1 + \mu_2, \sqrt{\sigma_1^2 + \sigma_2^2})$.

30. Obtain the mean of uniform distribution.

31. Define beta distribution. What are its properties?

(6 × 4 = 24 Marks)

Section D

Answer any ***two*** questions. Each question carries 15 marks.

32. (a) Obtain the m.g.f. of binomial distribution and hence find its mean and variance.

    (b) State and prove the additive property of Poisson distribution.

33. (a) A random variable X has density $f(x) = x + y, 0 < x < 1, 0 < y < 1$. Show that X and Y are independent.

    (b) Define gamma distribution and derive its mean and variance.

34. (a) Define normal distribution. Explain the important properties also.

    (b) If $X \sim N(\mu, \sigma^2)$, then find $P(X \leq 20)$ and $P(0 \leq X \leq 12)$.

35. (a) State and prove Chebyshev's Inequality.

    (b) If E(X) = 3, $E(X)^2 = 13$, use Chebyshev's Inequality to find a lower bound for $P(-2 < X < 8)$.

(2 × 15 = 30 Marks)

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