23P402 (Pages: 2) Name: ....

Reg.No: ....

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2025

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19P MTH4 C15 - ADVANCED FUNCTIONAL ANALYSIS

(Mathematics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Define point spectrum, continuous spectrum and residual spectrum of a bounden operator A.
2. Prove that point spectrum σ_p of a self adjoint operator A satisfies $\sigma_p(A) \subseteq \mathbb{R}$.
3. State second Hilbert Schmidt Theorem.
4. Let A be a self adjoint operator. Show that $-I\|A\| \leq A \leq I\|A\|$ for any self-adjoint operator A.
5. if $A \geq 0$ and $\langle Ax, x \rangle = 0$, then prove that $Ax = 0$
6. Let $E_i = \operatorname{Im} P_i$; i.e $E_i = P_i H$. If $P_1 P_2 = 0$, then prove that $P_2 P_1 = 0$, $E_1 \perp E_2$ and $P_1 + P_2$ is an orthoprojection onto $E_1 \oplus E_2$.
7. Define second category set with example.
8. With example define closed graph operator.

(8 × 1 = 8 Weightage)

Part B

Answer any ***two*** questions each unit. Each question carries 2 weightage.

UNIT - I

9. Show that for every $\varepsilon > 0$, there is only a finite number of linearly independent eigen vectors corresponding to eigen values λ_i with $|\lambda_i| \geq \varepsilon$.
10. State and Prove Fredholm's first theorem.
11. Prove that $\langle Ax, x \rangle \in \mathbb{R}$ for any $x \in H$ if and only if A is symmetric.

UNIT - II

12. Let $T : E \mapsto E$ be any linear operator, $E_1 + E_2 = E$ and let P be the projection onto E_1 parallel to E_2. Then prove that $PT = TP$ if and only if E_1 and E_2 are invariant subspaces of T
13. Let $Q_n(t)$ and $P_n(t)$ be sequences of polynomials. Assume that for all $t \in [m, M]$, $Q_n(t) \searrow \psi(t) \in K$ and $P_n(t) \searrow \varphi(t) \in K$.. Let $\psi(t) \leq \varphi(t)$ for all $t \in [m, M]$. Then prove that $\lim_{n\to\infty} Q_n(A) =: B_1 \leq B_2 := \lim_{n\to\infty} P_n(A)$.

14. Explain Hilbert Theorem.

UNIT - III

15. Define Minkowski functional. Prove that it is sublinear.

16. Define an Ideal. Prove that if $\mathcal{A}$ does not have any proper ideal, then $\mathcal{A}$ is a field.

17. Let $\mathcal{A}$ is a Banach Algebra. Prove that an element x is invertible if and only if x does not belong to any poper ideal.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. State and Prove first Hilbert-Schmidt theorem.

19. Let $A_0 \leq A_1 \leq \quad \leq A_n \leq\leq A$. Then show that there exists a strong limit of $(A_n)_n$. (i.e., there exists a bounded operator B and $A_n x \to Bx$ for all $x \in H$.

20. State and prove the Banach open mapping theorem.

21. State and prove Banach-Steinhaus theorem.

(2 × 5 = 10 Weightage)

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