25P153 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC22PMST1C01 - ANALYTICAL TOOLS FOR STATISTICS – I

(Statistics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part-A

Answer any ***four*** questions. Each question carries 2 weightage.

1. Prove that the function $f(x, y) = 2x^4 - 3x^2y + y^2$ has neither a maximum nor a minimum at the origin.
2. (a) Establish Taylor's theorem for a multivariable function.
   (b) Explain derivatives of a multivariable function.
3. Find the minimum value of $f(x, y) = x^2 + 5y^2 - 6x + 10y + 6$.
4. If $f(z) = u + iv$ is an analytic function in a domain D, prove that the curves $u = constant, v = constant$ form two orthogonal families.
5. State and prove Cauchy's theorem for analytic function.
6. What is a singular point? Explain different types of isolated singularities.
7. Find the residue of $\frac{1}{(z^2+1)^3}$ at $z = i$.

(4 × 2 = 8 Weightage)

Part-B

Answer any ***four*** questions. Each question carries 3 weightage.

8. Derive the polar form of Cauchy-Reimann equation.
9. What do you mean by Morera's theorem? Explain.
10. Evaluate $\int_0^{2\pi} \frac{d\theta}{a+b\,cos\theta}$.
11. Find the Laplace transform of
    a) $sin\sqrt{t}$
    b) $(t^2 + 1)^2$
    c) $\frac{e^{-at}t^{n-1}}{(n-1)!}$

12. Find the inverse Laplace transform of

a) $\frac{1}{s^2(s^2-a^2)}$

b) $\frac{1}{(s^2+1)^2}$

13. Find the Fourier series expansion of $f(x)$, where $f(x) = \begin{cases} \pi + x & -\pi < x < 0 \\ \pi - x & 0 < x < \pi \end{cases}$.

14. Find finite Fourier $sine$and $cosine$ transform of $f(x) = x^2$; $0 < x < 4$.

(4 × 3 = 12 Weightage)

Part-C

Answer any ***two*** questions. Each question carries 5 weightage.

15. Establish Poisson's integral formula.

16. State and prove Taylor's theorem.

17. State and prove the Cauchy-Residue theorem and evaluate $\int_c \frac{e^z}{z(z-1)^2} dz$.

18. Solve the differential equation by the method of laplace transform
$ty'' + (1-2t)y' - 2y = 0, y(0) = 1, y'(0) = 2$.

(2 × 5 = 10 Weightage)

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