25P154 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC22PMST1C02 - ANALYTICAL TOOLS FOR STATISTICS – II

(Statistics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part-A

Answer any ***four*** questions. Each question carries 2 weightage.

1. Explain vector space and its axioms.
2. Show that a set of non null vectors $\alpha_1, \alpha_2, \ldots \alpha_n$ orthogonal in pairs is necessarily independent.
3. Define a Hermitian matrix. Show that the characteristic root of a Hermitian matrix are real.
4. Reduce the following matrix in to normal form and hence find its rank. $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$.
5. Define Minimal polynomial. Find the minimal polynomial of $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$
6. Define non-homogeneous system of linear equations.
7. Prove that every quadratic form can be reduced to a form containing square terms by a non-singular linear transformation.

(4 × 2 = 8 Weightage)

Part-B

Answer any ***four*** questions. Each question carries 3 weightage.

8. if W_1 and W_2 are finite subspace of a vectorspace V then prove that $d(W_1 \cap W_2) = r + m + n$.
9. Let $F : R^4 \to R^3$ be the linear mapping defined by $F(x, y, z, t) = (x - y + z + t, 2x - 2y + 3z + 4t, 3x - 3y + 4z + 5t)$, find basis and dimension of the image of F.
10. Find inverse of matrix A. Given A= $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$.
11. What is diagonalizable matrices. Prove that A is diagonalizable matrix iff $(A - \lambda_i)$ is diagonalisable.

12. Find the spectral decomposition of matrix A= $\begin{pmatrix} 4 & 1 \\ 1 & 3 \end{pmatrix}$.

13. Define g-inverse. Prove that g-inverse always exist and is not unique.

14. Reduce the quadratic form $3x_1^2 + 2x_1x_2 + 2x_1x_3 + 4x_2x_3$ in to a canonical form by the method of orthogonal reduction.

(4 × 3 = 12 Weightage)

Part-C

Answer any ***two*** questions. Each question carries 5 weightage.

15. (i) Define transpose and trace of a matrix and its properties.

    (ii) Define unitary matrix and its properties.

    (iii) Find inverse of the matrix A by elementary transformation method.

    A= $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$

16. (a) Define Orthogonal matrix. Show that characteristic roots of an orthogonal matrix is either 1 or -1.

    (b) Prove that any two characteristic vectors corresponding to two distinct characteristic roots of an orthogonal matrix are orthogonal.

    (c) Prove that any 2 characteristic vectors corresponding to two distinct characteristic roots of an unitary matrix are orthogonal.

17. (a) If A is Hermitian show that the eigen values of A are real.

    (b) Show that the rank of idempotent matrix is equal to its trace.

    (c) Show that every characteristic root of an idempotent matrix is either 0 or 1.

18. (a) Explain Moore-Penrose inverse and its properties. Show that Moore-Penrose inverse is unique.

    (b) Find Moore-Penrose inverse of $\begin{bmatrix} 2 & 3 & 5 \\ 4 & 1 & 5 \\ 7 & 0 & 7 \\ 2 & 5 & 7 \end{bmatrix}$.

(2 × 5 = 10 Weightage)

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