25P155 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC22PMST1C03 - DISTRIBUTION THEORY

(Statistics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part-A

Answer any ***four*** questions. Each question carries 2 weightage.

1. If X is a random variable with cumulants K_r; $r = 1, 2, \ldots$ Find the cumulants of i) cX ii) $c + X$, where c is a constant.
2. Derive the probability density function of Poisson distribution.
3. Define log-normal distribution. Derive the median of log-normal distribution.
4. If X and Y are independent random variables with standard exponential distribution. Show that $Z = \frac{X}{Y}$ has an F-distribution.
5. Define bivariate normal distribution. If (X, Y) has a bivariate normal distribution, X and Y are independent if and only if $\rho = 0$.
6. Define a finite mixture of probability density function. Verify that a mixture of pdf's satisfies the properties of a pdf.
7. Define sampling distribution and standard error. Obtain standard error of mean when population is large.

(4 × 2 = 8 Weightage)

Part-B

Answer any ***four*** questions. Each question carries 3 weightage.

8. Let X be a random variable with p.m.f $P(X = j) = p_j,\ j = 0, 1, 2, \ldots$. Define $q_j = P(X > j)$, $j = 0, 1, 2, \ldots$ and $Q(s) = \sum_{j=0}^{\infty} q_j s^j$. Show that $Q(s) = \frac{1-P(s)}{1-s}$ for $|s| < 1$ where P(s) is the p.g.f of X.
9. Define hypergeometric distribution. Find its mean and variance.
10. Let $X_1, X_2, .., X_n$ be independent and identically distributed random variables with the uniform distribution on [0,1]. Find the distribution of k^{th} order statistic and identify the distribution.

11. Write down the differential equation satisfied by the Pearson system of distributions.What is the basis for classification of member of the family into various type? Give an example.

12. State Chebychev's inequality. If X be distributed with pdf $f(x) = 1,\ 0 < X < 1.$ prove that $P(|X - \frac{1}{2}| < 2\sqrt{\frac{1}{2}}) \geq \frac{3}{4}.$

13. Obtain the joint probability density function of r^{th} and s^{th} order statistic.

14. Deduce the moment generating function of non- central chi-square distribution.

(4 × 3 = 12 Weightage)

Part-C

Answer any ***two*** questions. Each question carries 5 weightage.

15. Define the power series distribution. Identify the members of the family. Also establish a recurrence relation satisfied by the cumulants of this family.

16. If X and Y are independent one parameter Gamma variates with parameters m and n respectively. Derive the distribution of U and V in the following cases and hence find their marginal distributions.
(i) $U = X + Y$ and $V = \frac{X}{X+Y}$
(ii) $U = X + Y$ and $V = \frac{X}{Y}$
(iii) $U = X + Y$ and $V = \frac{X-Y}{X+Y}$

17. Let X and Y be jointly distributed with pdf $f(x,y) = \begin{cases} \frac{1}{4}(1+xy) & |x|<1, |y|<1 \\ 0 & otherwise \end{cases}$. Show that X and Y are not independent but X^2 and Y^2 are independent.

18. Let X and Y be independent χ^2 random variables with m and n d.f respectively. Show that $U = X + Y$ and $V = \frac{X}{X+Y}$ are independent and obtain their distributions.

(2 × 5 = 10 Weightage)

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