25P156 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC22PMST1C04 - PROBABILITY THEORY

(Statistics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part-A

Answer any ***four*** questions. Each question carries 2 weightage.

1. Show that every monotone sequence of sets is convergent.
2. What is meant by independence of two random variables? Let $\{\Omega, \mathscr{A}, P\}$ be a probability space and A, B be two subsets of Ω. Define two random variables such that $X(\omega) = I_A(w)\ and\ Y(\omega) = I_B(w)$ Show that X and Y are independent
3. Define distribution function of a ransom variable. Examine whether the following is distribution function of a random variable or not .
$$F(x) = \begin{cases} 0, & \text{if } x \le 1 \\ 1 - \frac{1}{x}, & \text{if } x > 1 \end{cases}$$
4. Define expectation of a measurable function of X. If X is a random variable taking values 1,2,3,... $P(X = i) = p_i$, i=1,2,3,... show that $E(X) = \sum_{n=1}^{\infty} P(X \ge n)$.
5. Define convergence in rth mean. Show that the sequence $\{X_n, n \ge 1\}$ if converges in r^{th} mean, imples that $\{X_n\}$ converges in probability.
6. Define complete convergence of a seqence of distribution function $\{F_n, n \ge 1\}$. If $F_n(x) = \begin{cases} 0, \ if\ x < 0 \\ 1 - e^{-nx}, \ if\ x \ge 0 \end{cases}$ examine whether it is completely convergent or not.
7. Examine whether WLLN holds in the following sequence of independent random variables $P(X_k = \pm 2^k) = \frac{1}{2^{k+1}}\ and\ P(X_k = \pm 1) = \frac{1}{2}(1 - \frac{1}{2^k})$

(4 × 2 = 8 Weightage)

Part-B

Answer any ***four*** questions. Each question carries 3 weightage.

8. (a) Define conditional probability measure.

   (b) Define induced probability space.

9. Prove that $E|X+Y|^r \leq C_r E|X|^r + C_r E|Y|^r$, where $C_r = \begin{cases} 1, & \text{if } r \leq 1 \\ 2^{r-1}, & \text{if } r \geq 1 \end{cases}$

10. a) Let X be an integer valued random variable. Then show that its probability mass function is given by $p(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-itx}\phi(t)dt,\ x = 0, \pm1, \pm2, \pm3...$

    b) If the characteristic function of a random variable X is $\phi_x(t) = q + pe^{it}$ derive its probability mass function

11. State and prove Kolmogorov 0-1 law.

12. State and prove a necessary and sufficient condition for the convergence of a sequence random variables in probability to zero

13. Show that If $g(.)$ is uniformly continuous and bounded on R and $F_n(x) \xrightarrow{C} F(x)$ implies $\int_R gdF_n \to \int_R gdF$ as $n \longrightarrow \infty$

14. State and prove Kolmogorov inequality.

(4 × 3 = 12 Weightage)

Part-C

Answer any ***two*** questions. Each question carries 5 weightage.

15. Derive the characteristic function of a random variable X having probability density function as follows

    (i) $f(x) = \frac{1}{\pi(1+x^2)}, -\infty < x < \infty$

    (ii) $f(x) = \begin{cases} 1+x, & \text{if } -1 \leq x < 0 \\ 1-x, & \text{if } 0 < x \leq 1 \end{cases}$

16. If ν_r the rth absolute moment of $F(x)$ is finite, show that the characteristic function is differentiable r times. Converely if $\phi^r(0)$ exists and is finite, then show that $\nu_s < \infty$ for $s < r$ when r is odd and $s \leq r$ when r is even

17. State and prove Jordan decomposition theorem of distribution functions

18. State and prove Liapounov's form of central limit theorem.

(2 × 5 = 10 Weightage)

****