25P101 (Pages: 2) Name: …………………..

Reg. No: ………………..

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19PMTH1C01 – ALGEBRA - I

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Verify whether $\varphi(x,y) = (-y,x)$ is an isometry of the plane.
2. Describe the torsion subgroup of $Z_2 \times Z \times Z_4$.
3. Describe the center of every simple abelian group and the center of every non abelian group.
4. State Burnside's formula.
5. Write isomorphic refinements of the series $\{0\} < 10Z < Z$ and $\{0\} < 25Z < Z$.
6. Find the inverse of the word $1 + i + 2j + 2k$ in the ring of quaternions.
7. Define an ideal. Give an example of an ideal.
8. Prove that $x^3 + 3x + 2$ is irreducible over Z_5 .

(8 × 1 = 8 Weightage)

Part B

Answer any ***six*** questions from each unit. Each question carries 2 weightage.

Unit 1

9. Find all abelian groups up to isomorphism of order 720.
10. Prove that A_4 does not contain a subgroup of order 6.
11. Let X be a G-set and let $x \in X$. If $|G|$is finite, then prove that $|Gx|$is a divisor of $|G|$.

Unit 2

12. State and prove the third isomorphism theorem.
13. If p is a prime, then prove that every group of order p^2 is abelian.
14. Prove that every group G is the homomorphic image of a free group G.

Unit 3

15. Prove that the polynomial $x^2 - 2$ has no zeros in the rational numbers.

16. State and prove the Eisenstein criterion.

17. Draw the addition and multiplication tables for the group algebra Z_2G, where $G = \{e.a\}$ is a cyclic group of order 2?

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. a) Prove that $Z_m \times Z_n$ is isomorphic to Z_{mn} if and only if m and n are relatively prime.

    b) Prove that if m divides the order of a finite abelian group G, then G has a subgroup of order m.

19. a) Let H be a normal subgroup of a group G. Prove that the cosets of H forms a group G/H under the binary operation $(aH)(bH) = (ab)H$.

    b) Prove that the factor group of a cyclic group is cyclic.

    c) Show that if a finite group G contains a nontrivial subgroup of index 2 in G, then G is not simple.

20. a) If P_1 and P_2 are Sylow p-subgroups of a finite group G, then prove that P_1 and P_2 are conjugate subgroups of G.

    b) Let p and q be distinct primes with $p < q$. Prove that every group G of order pq is not simple.

21. Determine all groups of order 10 up to isomorphism.

(2 × 5 = 10 Weightage)

****