25P102 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19PMTH1C02 - LINEAR ALGEBRA

(Mathematics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part A

Answer any ***all*** questions. Each question carries 1 weightage.

1. Let S be a linearly independent subset of a vector space V .Suppose β is a vector in V which is not in the subsapce spanned by$S$.Then check whether the set obtained by adjoining β to S is linearly independent or not .
2. Let V be a vector space. Define an ordered basis for V. Give an example of an ordered basis for $\mathbb{R}^2$
3. Let T be a linear tranformation from V into V show that T is invertible implies T is non-singular.
4. Let $\mathcal{B} = \{\alpha_1, \alpha_2\}$ be the basis for $\mathbb{R}^2$ defined by $\alpha_1 = (2,3)$ and $\alpha_2 = (6,1)$. Find the dual basis of $\mathcal{B}$.
5. Define transpose of a linear transformation with example.
6. Let A and B be two matrices such that $B = P^{-1}AP$ then show that charaxteristic polynomial of A and B are same.
7. If V is an inner product space, the for any vectors α, β in V and any scalar c show that $||c\alpha|| = |c|||\alpha||$
8. Prove that an orthogonal set of non-zero vectors is linearly independent.

(8 × 1 = 8 Weightage)

Part B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT - I

9. Prove that the n tuple space F^n is a vector space.
10. Prove that the subspace spanned by a non-empty subset S of a vector space V is the set of all linear combinations of vectors in S.
11. Let V and W be vector spaces over the field F and let T be a linear transformation from Vin to W. Suppose that V is finite dimensional Then show that $\text{rank}(T) + \text{nullity}(T) = \dim V$

UNIT - II

12. Let T be the linear transformation from $\mathbb{R}^3$ into $\mathbb{R}^2$ defined by $T(x_1, x_2, x_3) = (x_1 + x_2, 2x_2 - x_3)$. IF $\mathcal{B} = \{\alpha_1, \alpha_2, \alpha_3\}$ and $\mathcal{B}' = \{\beta_1, \beta_2\}$, where $\alpha_1 = (1, 0, -1), \alpha_2 = (1, 1, 1), \alpha_3 = (1, 0, 0), \beta_1 = (0, 1), \beta_2 = (1, 0)$. Find the matrix of T relative to the pair $\mathcal{B}, \mathcal{B}'$.

13. If S is any subset of a finite dimensional vector space V, then prove that $(S^0)^0$ is the subspace spanned by S.

14. Let V be a finite dimensional vector space over F and let T be a linear operator on V.Show that T is diagonalizable implies the minimal polynomial for T has the form $p = (x - c_1)(x - c_2)\cdots(x - c_k)$ where $c_1, c_2, \ldots, c_k$ are distinct elements of F.

UNIT - III

15. Define projection on a vector space V. Prove that
    (i) Any projection E is diagonalizable.
    (ii) If E is projection on R along N, then $(I - E)$ is the projection on N along$R$.

16. State and Prove Polarization identities of an inner product.

17. Let V be an inner product space and S be any set of vectors in V. Define orthogonal complement of a set S. Prove that it is a subspace of V.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. If W_1 and W_2 are finite dimensional subspace of a vector space V then prove that $W_1 + W_2$ is finite dimensional. Also verify $\dim W_1 + \dim W_2 = \dim(W_1 \cap W_2) + \dim(W_1 + W_2)$.

19. Show that the dimesion of $L(V, V)$ is n^2 where V is finite dimesional vector space of dimension n

20. Let T be a linear operator on a finite dimensional vector space V. If $f$is the characteristic polynomial for T, then show that $f(T) = 0$.

21. (a) Show that the mapping $\beta \to \beta - E\beta$ is the orthogonal projection of V on $W^\perp$. where V is an inner product space, W a finite dimensional subspace, and E the orthogonal projection of V on W.
    (b) State and Prove Bessel's Inequality.

(2 × 5 = 10 Weightage)

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