25P103 (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19PMTH1C03 - REAL ANALYSIS - I

(Mathematics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Prove that countable union of countable sets is countable.
2. Is union of an infinite collection of closed sets closed. Justify your answer.
3. If X and Y be metric spaces and $E \subset X$, f maps E into Y and p is a limit point of E. If f has a limit at p, prove that this limit is unique.
4. Let E be noncompact set in $\mathbb{R}$. Prove that there exists a continuous and bounded function on E, which has no maximum.
5. Verify whether mean value holds for the function $f(x) = e^{ix}, x \in [0, 2\pi]$.
6. Let $\{f_n(x)\}$ be a sequence of functions defined on E such that $|f_n(x)| \leq M_n$, $x \in E, n = 1, 2, 3, \ldots$. Then prove that $\sum f_n$ converges uniformly on E, if $\sum M_n$ converges.
7. Prove that a sequence $\{f_n\}$ converges to f with respect to the metric of $\mathcal{C}(X)$, the set of all complex-valued, continuous, bounded functions with domain X, if and only if $\{f_n\}$ converges to f uniformly on X.
8. Prove that every member of an equicontinuos family of functions is uniformly continous.

(8 × 1 = 8 Weightage)

Part B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT - I

9. Let $Y \subset X$. Prove that a subset E of Y is open relative to Y if and only if $E = Y \cap G$ for some open subset G of X.
10. Prove that every K cell is compact.
11. Prove that monotonic functions have no discontinuities of the second kind.

UNIT - II

12. State and prove mean value theorem.

13. Show that if f is monotonic on $[a, b]$ and α is continuous on $[a, b]$, then $f \in \mathscr{R}(\alpha)$ on $[a, b]$.

14. State and prove fundamental theorm of calculus

UNIT - III

15. Show that a continuously differentiable curve γ defined on an interval $[a, b]$ is rectifiable and $\Lambda(\gamma) = \int_a^b |\gamma'(t)| dt$

16. Prove that the series $\sum_{n=1}^{\infty} (-1)^n \frac{x^2+n}{n^2}$ converges uniformly in every bounded interval.

17. If $\{f_n\}$ is a sequence of continuous functions on a set E, and if $f_n \to f$ uniformly on E, then prove that f is continuous on E.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Let f be a mapping from a metric space X into a metric space Y. Show that
(a) f is continuous on X if and only $f^{-1}(C)$ is closed in X for every closed set C in Y.
(b) If f is continuous on X, then f^{-1} is continuous on Y.

19. (a) State and Prove L'Hospital's rule.
(b) Iff is differentiable on $[a, b]$, then prove that f' cannot have any simple discontinuities on $[a, b]$.

20. (a) State and prove Taylor's theorem.
(b) Prove that the function $f \in \mathscr{R}(\alpha)$ on $[a, b]$ if and only if for every $\epsilon > 0$, there exists a partition P of
$[a, b]$ such that $U(P, f, \alpha) - L(P, f, \alpha) < \epsilon$.

21. (a) If $f_1, f_2 \in \mathscr{R}(\alpha)$ on $[a, b]$, then prove that $(f_1 + f_2) \in \mathscr{R}(\alpha)$ on $[a, b]$ and
$$\int_a^b (f_1 + f_2)\ d\alpha = \int_a^b f_1\ d\alpha + \int_a^b f_2\ d\alpha.$$
(b) If $f \in \mathscr{R}(\alpha)$ on $[a, b]$ and c is any constant, then prove that $cf \in \mathscr{R}(\alpha)$ on $[a, b]$ and
$$\int_a^b cf\ d\alpha = c\int_a^b f\ d\alpha.$$

(2 × 5 = 10 Weightage)

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