24P303 (Pages: 2) Name: ....

Reg.No: ....

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19PMTH3C13 - FUNCTIONAL ANALYSIS

(Mathematics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Define seminorm in a linear space E
2. Define Cauchy sequence in a normed space X. Prove that if x_n is a Cauchy sequence, then prove that it is a bounded sequence.
3. Define an inner product on $\mathbb{C}$ and show that it is an inner product.
4. Show that if f_i is a complete system in a Hilbert space H and $x \perp f_i$, then $x = 0$.
5. Show that for every closed subspace of H,$L \oplus L^{\perp} = H$
6. Show that f is a bounded functional if and only if f is a continuous functional.
7. State Arzela theorem.
8. Define norm convergence, strong convergence and weak convergence.

(8 × 1 = 8 Weightage)

Part B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT - I

9. Show that the dimension of E/E_1 is n if and only if there exist $x_1, x_2, \ldots, x_n$ linearly independent vectors relative to E_1 such that for every $x \in E$ there exist unique set of numbers $a_1, a_2, \ldots a_n$ and a unique vector $y \in E_1$ such that $x = \sum_{i=1}^{n} a_i x_i + y$
10. Define a norm on $c[0, 1]$ and prove that it is a norm.
11. Let O be an openset then prove that $F = O^c$ closed. Also prove if F is closed set then F^c is open.

UNIT - II

12. State and Prove Bessel's inequality.

13. Let M be a convex closed set in H. Show that there exists a unique $y \in M$ such that $\rho(x, M) = \|x - y\|$

14. Consider $f \in E^{\#} - \{0\}$. Then show that
(a) Codim ker $f = 1$
(b) If $f, g \in E^{\#} - \{0\}$ and ker $f =$ ker g, then there exists $\lambda \neq 0$ such that $\lambda f = g$.

UNIT - III

15. Prove that for any normed space X, the dual space X^* is always complete.

16. Prove that the dual space of c_0 is l_1.

17. If $\|A\| = q < 1$, then prove that $(I - A)$ is invertible and $(I - A)^{-1} = \sum_0^\infty A^k$. More over $\|(I - A)^{-1}\| \leq \dfrac{1}{1 - \|A\|}$.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. State and Prove Minkowski's ineqality for sequences.

19. (a) Prove that a complete orthonormal system $\{e_i\}_{i=1}^\infty$ in H is a basis in H.
(b) State and Prove Parseval's Identity.

20. State and prove Riesz representation theorem.

21. Prove that M is relatively compact if and only if for every $\varepsilon > 0$, there exists a finite ε- net in M.

(2 × 5 = 10 Weightage)

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