24P304 (Pages: 2) Name……………………..

Reg. No…………………..

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19P MTH3 C14 – PDE AND INTEGRAL EQUATIONS

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Explain the method of characteristics to solve a first order linear partial differential equation.
2. Let $u(x,t)$ be a solution of the wave equation $u_{tt} - c^2 u_{xx} = 0$, which is defined in the whole plane. If u is constant on the line $x = 2 + ct$, then prove that $u_t + c\, u_x = 0$.
3. State the heat conduction problem for a rod of finite length and find its generalized solution.
4. Show that the Dirichlet problem in a bounded domain D has atmost one solution in $C^2(D) \cap C(\overline{D})$.
5. If $u(x,y)$ is a harmonic function in a domain D, then show that $u \in C^\infty(D)$.
6. If $y''(x) = F(x)$ and y satisfies the end conditions $y(0) = 0$ and $y(1) = 0$, then show that $y(x) = \int_0^x (x-\xi)F(\xi)d\xi - x\int_0^1 (1-\xi)F(\xi)d\xi$.
7. Show that the characteristic numbers of a homogeneous Fredholm integral equation with a real symmetric kernel are all real.
8. Obtain the resolvent kernel associated with the kernel $K(x,\xi) = 1 - 3x\xi$ in $(0,1)$.

(8 × 1 = 8 Weightage)

PART B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT - I

9. Solve $-yu_x + xu_y = 0,\ u(x,0) = sinx$ for $x > 0$ by Lagrange method.
10. Reduce to canonical form, the PDE $x^2u_{xx} - 2xyu_{xy} + y^2u_{yy} + xu_x + yu_y = 0$.
11. Solve the problem

$$u_{tt} - u_{xx} = t^7 \qquad -\infty < x < \infty,\ t > 0,$$
$$u(x,0) = 2x + sinx \qquad -\infty < x < \infty$$
$$u_t(x,0) = 0 \qquad -\infty < x < \infty.$$

UNIT - II

12. Show that the solution to the following problem is unique:

$$u_{tt} - c^2 u_{xx} = F(x,t) \qquad 0 < x < L,\ t > 0,$$

$$u_x(0,t) = a(t),\ u_x(L,t) = b(t) \qquad t \geq 0,$$

$$u(x,0) = f(x),\ u_t(x,0) = g(x) \qquad 0 \leq x \leq L\ .$$

13. State and prove the strong maximum principle.

14. Solve the Laplace equation in the square $0 < x,\ y < \pi$ subject to the Dirichlet condition $u(x,0) = 1984,\ u(x,\pi) = u(0,y) = u(\pi,y) = 0.$

UNIT - III

15. Using Green's function, transform the differential equation

$\frac{d^2y}{dx^2} + y = x,\ \ y(0) = y(1) = 0$ to an integral equation.

16. Find the general solution of the equation $y(x) = \lambda \int_0^{2\pi} \sin(x+\xi)\, y(\xi)\, d\xi + x$.

17. Describe the iterative procedure to solve a Fredholm integral equation of second kind.

(6 × 2 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

18. By the characteristics method, solve $-yu_x + xu_y = u$ subject to the initial condition $u(x,0) = \psi(x)$.

19. Fix $T > 0$. Show that for $0 \leq t \leq T$, the Cauchy problem

$$u_{tt} - c^2 u_{xx} = F(x,t) \qquad -\infty < x < \infty,\ t > 0,$$

$$u(x,0) = f(x), \qquad u_t(x,0) = g(x) \qquad -\infty < x < \infty$$

is well posed if $F,\ F_x \in C(\mathbb{R}^2),\ f \in C^2(\mathbb{R}),\ g \in C^1(\mathbb{R})$.

20. Show that the solution for the Dirichlet problem for a circle of radius a is given by the Poisson integral formula.

21. Make use of an appropriate formula to show that the iterative procedure will converge if $|\lambda| < 3$ for the integral equation $y(x) = 1 + \lambda \int_0^1 x\xi\, y(\xi) d\xi$ and solve it.

(2 × 5 = 10 Weightage)

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