24P301 (Pages: 2) Name……………………..

Reg. No…………………..

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2025

(CBCSS-PG)

(Regular/Supplementary/Improvement)

CC19PMTH3C11 – MULTIVARIABLE CALCULUS AND GEOMETRY

(Mathematics)

(2019 Admission onwards)

Time: Three Hours Maximum: 30 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Suppose X is a vector space and $\dim X = n$. Show that a set E of n vectors in X spans X if and only if E is independent.
2. Suppose f is a $\mathcal{C}'$- mapping of an open set $E \subset R^n$ into R^n and $f'(x)$ is invertible for every $x \in E$. Show that f is an open mapping of E into R^n.
3. Prove that a linear operator A on R^n is invertible if and only if $\det[A] \neq 0$.
4. If the tangent vector of a parametrized curve is constant, then show that the image of the curve is (part of) a straight line.
5. Find the signed curvature of the catenary $\gamma(t) = (t, cosh\, t)$.
6. Let $f: S_1 \to S_2$ be a diffeomorphism. If σ_1 is an allowable surface patch on S_1, then show that $f \circ \sigma_1$ is an allowable surface patch on S_2.
7. Show that the level surface $\{(x, y, z) \in R^3 \mid x^2 + y^2 + z^2 = 1\}$ is a smooth surface.
8. Show that the second fundamental form is symmetric.

($8 \times 1 = 8$ Weightage)

PART B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT I

9. Suppose a vector space X is spanned by a set of r vectors. Show that $\dim X \leq r$.
10. Let Ω be the set of all invertible linear operators on R^n. If $A \in \Omega$, $B \in L(R^n)$, and $\| B - A \| \| A \| < 1$, then show that $B \in \Omega$. Also show that Ω is an open subset of $L(R^n)$.
11. Suppose f maps an open set $E \subset R^n$ into R^m and assume $f \in \mathcal{C}'(E)$. Show that the partial derivatives $D_j f_i$ exist and are continuous on E for $1 \leq i \leq m, 1 \leq j \leq n$.

UNIT II

12. Show that unit-speed reparametrization of a regular closed curve is closed.
13. Compute the curvature of the circular helix $\gamma(\theta) = (a\cos\theta, a\sin\theta, b\theta), \theta \in R$, where a and b are constants.
14. Show that if $f(x, y)$ is a smooth function, its graph $\{ (x, y, z) \in R^3 : z = f(x, y)\}$ is a smooth surface.

UNIT III

15. Compute the second fundamental form of $\sigma(u, v) = (\cos\theta\cos\varphi, \cos\theta\sin\varphi, \sin\theta)$.
16. i) Show that the area of a surface patch is unchanged by reparametrization.

    ii) Show that $\| \sigma_u \times \sigma_v \| = (EG - F^2)^{\frac{1}{2}}$.
17. Show that the normal curvature of any curve on a sphere of radius r is $\pm \frac{1}{r}$.

(6 × 2 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

18. State and prove the implicit function theorem.
19. i) Show that a parametrized curve has a unit-speed reparametrization if and only if it is regular.

    ii) Let γ be a unit-speed curve in R^3 with constant curvature and zero torsion. Show that γ is a parametrization of (part of) a circle.
20. Let S and $\tilde{S}$ be surfaces and let $f : S \to \tilde{S}$ be a smooth map. Show that f is a local diffeomorphism if and only if, for all $p \in S$, the linear map $D_p f : T_pS \to T_{f(p)} \tilde{S}$ is invertible.
21. i) Calculate the principal curvatures of the catenoid

    $\sigma(u, v) = (\cosh u \cos v, \cosh u \sin v, u)$.

    ii) Let σ be a surface patch of an oriented surface S. Show that the matrix of the Weingarten map of S at p with respect to the basis $\{\sigma_u, \sigma_v\}$ of T_pS is $\mathcal{F}_I^{-1}\mathcal{F}_{II}$, where $\mathcal{F}_I = \begin{bmatrix} E & F \\ F & G \end{bmatrix}$ and $\mathcal{F}_{II} = \begin{bmatrix} L & M \\ M & N \end{bmatrix}$.

(2 × 5 = 10 Weightage)

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