25U107 (Pages: 2) Name : .....

Reg. No : .....

FIRST SEMESTER SEMESTER UG DEGREE EXAMINATION, NOVEMBER 2025

(FYUGP)

(Regular/Supplementary/Improvement)

CC24UMAT1CJ101 - DIFFERENTIAL CALCULUS

(B.Sc. Mathematics - Major Course)

(2024 Admission onwards)

Time: 2.0 Hours Maximum: 70 Marks

Credit: 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 3 marks.

1. Does the function $f(x) = \sqrt{4 - x^2}$ defined in the interval [-2,2] has a left hand limit at $x = -2$ and a right hand limit at $x = 2$. [Level:2] [CO1]

2. Check whether the function $x^4 + 3x$ is even, odd or neither. Justify your answer. [Level:2] [CO1]

3. Discuss the behaviour of $f(x)$ as $x \longrightarrow 0$ [Level:2] [CO1]
   where $f(x) = \begin{cases} 0, & x \leq 0 \\ sin\frac{1}{x}, & x > 0. \end{cases}$

4. What is implicit differentiation? Use implicit differentiation to find $\frac{dy}{dx}$ if $y^2 = x$. [Level:3] [CO3]

5. Define average rate of change and instantaneous rate of change of a function $f(x)$. [Level:2] [CO2]

6. State the intermediate value theorem. [Level:1] [CO2]

7. State the constant multiple rule and find the deivative of $3x^2$. [Level:2] [CO2]

8. Define increasing and decreasing functions [Level:1] [CO4]

9. Explain second derivative test for concavity? [Level:1] [CO5]

10. Find $\lim\limits_{x \to \infty} \dfrac{5x^2 + 8x - 3}{3x^2 + 2}$ [Level:2] [CO5]

(Ceiling: 24 Marks)

Part B (Paragraph questions/Problem)

Answer ***all*** questions. Each question carries 6 marks.

11. a) Apply the properties of limits to evaluate $\lim\limits_{x \to 4} \dfrac{4x - x^2}{2 - \sqrt{x}}$. [Level:3] [CO1]

    b) If $2 - x^2 \leq g(x) \leq 2\cos x$, find $\lim\limits_{x \to 0} g(x)$.

12. Find the center and radius of the circles $x^2 + y^2 - 8x + 4y + 16 = 0$ and $x^2 + y^2 - 3y - 4 = 0$. [Level:2] [CO1]

13. State chain rule and use it to find the derivative of $g(t) = tan(5 - sin2t)$. [Level:3] [CO3]

14. Using definition of derivative, differentiate $y = \sqrt{x}$ for $x > 0$. Find the tangent line to the curve $y = \sqrt{x}$ at $x = 4$. [Level:3] [CO2]

15. Compute the absolute maximum and minimum values of $F(x) = -\frac{1}{x}, \quad -2 \leq x \leq -1$ [Level:3] [CO4]

16. Apply the Mean Value Theorem to the function $f(x) = \sqrt{x-1}$ in the interval $[1, 3]$ and determine the value or values of *c* that satisfy the theorem. [Level:3] [CO4]

17. Find the asymtotes of the curve $y = 2 + \frac{\sin x}{x}$. [Level:2] [CO5]

18. Identify the inflection points and local maxima and minima of the function $y = x^3 + 3x^2 - 4$ and also identify the intervals on which the functions are concave up and concave down. [Level:3] [CO5]

(Ceiling: 36 Marks)

Part C (Essay questions)

Answer any ***one*** question. The question carries 10 marks.

19. (a) Explain first derivative test for local minimum. [Level:3] [CO4]

    (b) Find the critical points of $f(x) = x^{1/3}(x-4) = x^{4/3} - 4x^{1/3}$. Identify the intervals on which f is increasing and decreasing. Find the function's local and absolute extreme values.

20. Graph the function $y = \frac{x^3+1}{x}$ with asymptotes and dominant terms. [Level:3] [CO5]

(1 × 10 = 10 Marks)

****