**25U110** (Pages: 2) Name : ....................................

Reg. No : ....................................

**FIRST SEMESTER UG DEGREE EXAMINATION, NOVEMBER 2025**

(FYUGP)

(Regular/Supplementary/Improvement)

**CC24UMAT1CJ102 - ELEMENTARY NUMBER THEORY**

(B.Sc. Mathematics - Major Course)

(2024 Admission onwards)

Time: 2.0 Hours Maximum: 70 Marks

Credit: 4

**Part A** (Short answer questions)

Answer ***all*** questions. Each question carries 3 marks.

1. Find the $gcd(72, 140)$. [Level:2] [CO1]
2. Find the quotient and remainder when $78$ is divided by $11$. [Level:2] [CO1]
3. Explain the sieve of Eratosthenes [Level:2] [CO2]
4. Is $1076x + 2076y = 1155$, Diophantine equation is solvable [Level:2] [CO2]
5. Prove that if $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$, then $a \equiv c \pmod{n}$. [Level:2] [CO3]
6. Define congruence and give an example [Level:2] [CO3]
7. For each positive integer $n \geq 1$, show that $n = \sum_{\frac{d}{n}} \phi(d)$. [Level:2] [CO5]
8. Define Euler Phi function. Find $\phi(8)$. [Level:2] [CO5]
9. State Euler's theorem. [Level:2] [CO5]
10. When we say that the quadratic equation $x^2 + 1 \equiv 0 \pmod{p}$, where $p$ is an odd prime, has a solution? [Level:1] [CO4]

**(Ceiling: 24 Marks)**

**Part B** (Paragraph questions/Problem)

Answer ***all*** questions. Each question carries 6 marks.

11. If $gcd(a, b) = 1$ and $gcd(a, c) = 1$, then prove that $gcd(a, bc) = 1$. [Level:2] [CO1]
12. By induction prove that $21|(4^{n+1} + 5^{2n-1})$. [Level:2] [CO1]
13. State and prove Wilson's theorem. [Level:1] [CO4]
14. Prove that the only prime of the form $n^3 - 1$ is $7$. [Level:3] [CO2]

15. Prove that $17^{\#} + 1$ is not prime where $p^{\#}$ is he product of all primes that are less than or equal to $p$. [Level:2] [CO2]

16. Show that the linear congruence $ax \equiv b \pmod{n}$ has a solution if and only if $d/b$, where $d = gcd(a, n)$. If $d/b$, then it has d mutually incongruent solutions. [Level:2] [CO3]

17. State and prove Fermat's theorem. [Level:2] [CO3]

18. 1. Evaluate $\phi(5040)$ [Level:3] [CO5]
    2. Evaluate $\phi(36000)$

**(Ceiling: 36 Marks)**

**Part C** (Essay questions)

Answer any ***one*** question. The question carries 10 marks.

19. Show that the linear diophantine equation $ax + by = c$ has a solution iff $d/c$, where $d = gcd(a, b)$. If $x_0, y_0$ is any particular solution of this equation, then all the other solutions are given by $x = x_0 + (\frac{b}{d})t, y = y_0 - (\frac{a}{d})t$. [Level:2] [CO2]

20. Solve each of the set of simultaneous congruences: [Level:3] [CO3]
    (i) $x \equiv 5 \pmod{6}, x \equiv 4 \pmod{11}, x \equiv 3 \pmod{17}$
    (ii) $x \equiv 1 \pmod{3}, x \equiv 2 \pmod{5}, x \equiv 3 \pmod{7}$

**(1 × 10 = 10 Marks)**

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