25U109 (Pages: 2) Name : .....

Reg. No : .....

FIRST SEMESTER UG DEGREE EXAMINATION, NOVEMBER 2025

(FYUGP)

(Regular/Supplementary/Improvement)

CC24UMAT1MN102 - CALCULUS OF SINGLE VARIABLE

(Mathematics - Minor Course)

(2024 Admission onwards)

Time: 2.0 Hours Maximum: 70 Marks

Credit: 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 3 marks.

1. Find $\lim_{x\to+\infty} \dfrac{7-6x^5}{x+3}$. [Level:3] [CO1]

2. Evaluate $\lim_{x\to a} \dfrac{-1}{(x-a)^2}$ using one sided limits. [Level:3] [CO1]

3. Determine whether the function $g(x) = \begin{cases} \frac{x^2-4}{x-2}, & x \neq 2 \\ 3, & x = 2 \end{cases}$ is continuous at x = 2. [Level:2] [CO1]

4. Find $f'(x)$ if $f(x) = 4\cos x + 2\sin x$. [Level:2] [CO2]

5. Find $f'(x)$ if $f(x) = (x^3+2x)^{37}$. [Level:2] [CO2]

6. Find the slope of the tangent lines to the curve $y = \sqrt{x}$ at $x_0 = 4$ and $x_0 = 9$. [Level:2] [CO2]

7. Use L'Hospital's Rule to evaluate $\lim_{x\to 0}(\dfrac{1}{x} - \dfrac{1}{e^x-1})$. [Level:3] [CO4]

8. Find $\frac{dy}{dx}$ if $y = \sin^{-1} x + \cos^{-1} x$. [Level:2] [CO3]

9. Define inflection Point [Level:1] [CO5]

10. Find the critical points of $f(x) = x^3 - 3x + 1$. [Level:2] [CO5]

(Ceiling: 24 Marks)

Part B (Paragraph questions/Problem)

Answer ***all*** questions. Each question carries 6 marks.

11. Find a value of the constant k, if possible, that will make the functions continuous everywhere. [Level:3] [CO1]

$$(a) f(x) = \begin{cases} 7x-2, & x \leq 1 \\ kx^2, & x > 1 \end{cases}$$

$$(b) g(x) = \begin{cases} kx^2, & x \leq 2 \\ 2x+k, & x > 2 \end{cases}$$

12. Let $f(x) = \frac{\sqrt{x}-3}{2x-18}$ and $g(x) = \frac{x-9}{2\sqrt{x}-6}$. Compute $\lim_{x\to 9}[f(x) - 5g(x)]$. [Level:3] [CO1]

13. Evaluate (a) $\lim_{y\to -1} \dfrac{y^2-1}{y+1}$ (b) $\lim_{x\to -1} \dfrac{x^2+6x+5}{x^2-3x-4}$ (c) $\lim_{x\to 0} \dfrac{x^2-2x}{x}$ [Level:3] [CO1]

14. State sum and difference rule and use it to find $y = 3x^8 - 2x^5 + 6x + 1$. [Level:3] [CO2]

15. Use quotient rule to find $y'(x)$ for $y = \frac{x^3+2x^2-1}{x+5}$. [Level:3] [CO2]

16. Use logarithmic differentiation to find the derivative of $y = \frac{x^2\sqrt[3]{7x-14}}{(1+x^2)^4}$. [Level:3] [CO3]

17. Use L'Hospital's Rule to evaluate (a) $\lim_{x\to 2} \dfrac{x^2-4}{x^2+2x-8}$ (b) $\lim_{x\to +\infty} \dfrac{2x-5}{3x+7}$. [Level:3] [CO4]

18. Determine the intervals on which $f(x) = x^4 - 5x^3 + 9x^2$ is increasing and decreasing. [Level:3] [CO5]

(Ceiling: 36 Marks)

Part C (Essay questions)

Answer any ***one*** question. The question carries 10 marks.

19. (a) Use implicit differentiation to find $\frac{dy}{dx}$ for the Folium of Descartes $x^3 + y^3 = 3xy$. [Level:3] [CO3]

(b) Find an equation for the tangent line to the Folium of Descartes at the point $(\frac{3}{2}, \frac{3}{2})$.

(c) At what point in the first quadrant is the tangent line to the Folium of Descartes horizontal?

20. Sketch the graph of the equation $f(x) = x^2 - 3x - 4$ and identify the locations of the intercepts, relative extrema, and inflection points. [Level:3] [CO5]

(1 × 10 = 10 Marks)

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