25U108 (Pages: 2) Name : ....

Reg. No : ....

FIRST SEMESTER SEMESTER UG DEGREE EXAMINATION, NOVEMBER 2025

(FYUGP)

(Regular/Supplementary/Improvement)

CC24UMAT1MN105 - MATRIX THEORY

(Mathematics - Minor Course)

(2024 Admission onwards)

Time: 2.0 Hours Maximum: 70 Marks

Credit: 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 3 marks.

1. Solve the linear system $\begin{aligned} x - y &= 1 \\ 2x + y &= 6 \end{aligned}$ [Level:2] [CO1]

2. If $A = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.Find $5A + 3B$ and $5A + B^T$ [Level:2] [CO1]

3. Give an example to show that matrix multiplication is not commutative [Level:2] [CO1]

4. Verify that AA^T is symmetric where $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ [Level:2] [CO2]

5. Determine conditions on the $b_i s$if any, in order to guarantee that the linear system is consistent. [Level:3] [CO2]
$$\begin{aligned} x_1 + 3x_2 &= b_1 \\ -2x_1 + x_2 &= b_2 \end{aligned}$$

6. Verify that $(A^T)^{-1} = (A^{-1})^T$ where $A = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}$ [Level:2] [CO2]

7. Verify $det(A) = det(A^T)$ by considering $A = \begin{bmatrix} -6 & 1 \\ 2 & -2 \end{bmatrix}$ [Level:2] [CO3]

8. Find $\bar{u} \times \bar{v}$ where $\bar{u} = (1, 2, -2)$ and $\bar{v} = (3, 0, 1)$ [Level:2] [CO5]

9. Let $\bar{u} = (1, 2, -3, 5, 0), \bar{v} = (0, 4, -1, 1, 2)$ and $\bar{w} = (7, 1, -4, -2, 3)$. Find the components of $3(2\bar{u} - \bar{v})$ and $\frac{1}{2}(\bar{w} - 5\bar{v} + 2\bar{u}) + \bar{v}$ [Level:2] [CO5]

10. Find $\|\text{proj}_{\bar{a}}\bar{u}\|$ where $\bar{u} = (6, 2)$ and $\bar{a} = (3, -9)$ [Level:2] [CO5]

(Ceiling: 24 Marks)

Part B (Paragraph questions/Problem)

Answer ***all*** questions. Each question carries 6 marks.

11. Using elementary row operations solve the system. [Level:2] [CO1]

$2x - y = -2$
$3x + 4y = 3$

12. If $A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$ Show that $(A^{-1})^3 = (A^3)^{-1}$ [Level:3] [CO2]

13. Verify that $(AB)C = A(BC)$ for the matrices [Level:2] [CO2]

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$

14. Use the inversion algorithm to find A^{-1} if it exists $A = \begin{bmatrix} 1 & -5 \\ 3 & -16 \end{bmatrix}$. [Level:3] [CO2]

15. Using Cramer's rule solve [Level:3] [CO4]

$$\begin{aligned} 7x_1 - 2x_2 &= 3 \\ 3x_1 + \ \ x_2 &= 5 \end{aligned}$$

16. Given $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 3 \\ 5 & 8 \end{bmatrix}$. Show that $det(AB) = det(A)det(B)$. Also determine whether $det(A + B) = det(A) + det(B)$ [Level:3] [CO4]

17. Find vector and parametric equations of the plane that passes through the point $P_0(-3, 1, 0)$ and is parallel to the vectors $\bar{v}_1 = (0, -3, 6)$ and $\bar{v}_2 = (-5, 1, 2)$ [Level:3] [CO5]

18. Find the Euclidean distance between $\bar{u} = (3, 3, 3)$ and $\bar{v} = (1, 0, 4)$. Also find the cosine of the angle between these vectors. State whether that angle is acute, obtuse,or right angled. [Level:3] [CO5]

(Ceiling: 36 Marks)

Part C (Essay questions)

Answer any ***one*** question. The question carries 10 marks.

19. [Level:2] [CO1]

a. Solve the linear system by Gaussian elimination $\begin{aligned} 2x_1 + 2x_2 + 2x_3 &= 0 \\ -2x_1 + 5x_2 + 2x_3 &= 1 \\ 8x_1 + x_2 + 4x_3 &= -1 \end{aligned}$

b. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ Show that $(A^{-1})^T = (A^T)^{-1}$

20. Find adjoint of the matrix $\begin{bmatrix} 2 & 0 & 0 \\ 8 & 1 & 0 \\ -5 & 3 & 6 \end{bmatrix}$ Determine whether the matrix is invertible, and if so, use the adjoint method to find its inverse. [Level:3] [CO3]

(1 × 10 = 10 Marks)

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