25U113S (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2025

(CBCSS - UG)

CC19UMTS1B01 / CC20UMTS1B01 - BASIC LOGIC AND NUMBER THEORY

(Mathematics - Core Course)

(2019 to 2023 Admissions - Supplementary)

Time : 2.5 Hours Maximum : 80 Marks

Credit : 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Evaluate the boolean expression, where $a = 3, b = 5$ and $c = 6$.
   $[\sim (a > b)] \wedge (b < c)$
2. Define contrapositive and give example.
3. Define uniqueness quantifiers and give example.
4. Write
   (a) Addition law
   (b) Hypothetical syllogism.
5. Compute the first four terms of the sequence defined recursively : $a_1 = 1, a_n = a_{n-1} + 3$
6. Find the five consecutive composite numbers less than 100.
7. Are the integers 6 and 35 relatively prime?
8. State Dirichlet's Theorem
9. Find the remainder when $1! + 2! + 3! + \cdots + 100!$ is divisible by 15.
10. Prove or disprove $78 \equiv 48(mod 5)$ can be reduced to $13 \equiv 8(mod 5)$.
11. Determine the number of incongruent solutions of the linear congruence $28u \equiv 119(mod 91)$
12. Using divisibility test determine whether 1928 and 388 are divisible by 11.
13. Without using Wilsons theorem verify that $(p-1)! \equiv -1(mod p)$ for $p = 13$.
14. Let p be a prime and a any integer such that p does not divide a. Then show that a^{p-2} is an inverse of a modulo p.
15. Define Euler's phi function and compute $\phi(11)$.

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Verify $p \to (q \vee r) \equiv (p \wedge \sim q) \to r$.

17. Using mathematical induction prove that $\sum_{i=1}^{n} i^2 = \dfrac{n(n+1)(2n+1)}{6}$.

18. Find the number of positive integers less than or equal to 3000 and divisible by 3, 5, or 7.

19. Prove that every composite number n has a prime factor less than or equal to $\lfloor \sqrt{n} \rfloor$.

20. Using the Euclidean Algorithm, Express (3076, 1776) as a linear combination of 3076 and 1776.

21. Prove that if p be a prime and $p|a_1 a_2 \ldots . a_n$, where $a_1, a_2, \ldots . . a_n$ are positive integers, then $p|a_i$ for some i, where $1 \leq i \leq n$.

22. Using canonical decompositions, find the gcd of each pair; 72, 108.

23. State and prove Fermat's Little Theorem.

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. Explain Methods of Proof.

25. If a cock is worth fivecoins, a hen three coins, and three chicks together one coin, how many cocks, hens, and chicks, totaling 100, can be bought for 100 coins?

26. (a) Using inverses, find the incongruent solution of $48x \equiv 39(mod 17)$.

    (b) Using congruences solve $48x + 84y = 144$.

27. a) Using Euler's theorem find the remainder when 245^{1040} is divided by 18.

    b) Solve the linear congruence $7x \equiv 8(mod 10)$.

(2 × 10 = 20 Marks)

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