24U312 (Pages: 3) Name : ....

Reg. No : ....

THIRD SEMESTER UG DEGREE EXAMINATION, NOVEMBER 2025

(FYUGP)

CC24UMAT3CJ202 - MATRIX ALGEBRA

(B.Sc. Mathematics / Mathematics & Computer Science Double Main - Major Course)

(2024 Admission - Regular)

Time: 2.0 Hours Maximum: 70 Marks

Credit: 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 3 marks.

1. Check whether the following equations are linear or not: [Level:2] [CO1]

   $a) 4x_1 - 5x_2 + 2 = x_1$

   $b) x_1 + 7x_2 = x_1 x_2$

2. Find the general solution of the linear system whose augmented matrix is $\begin{bmatrix} 0 & 1 & -6 & 5 \\ 1 & -2 & 7 & -6 \end{bmatrix}$ [Level:2] [CO1]

3. List five vectors in span$\{\mathbf{u}, \mathbf{v}\}$, where $\mathbf{u}=\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$ and $\mathbf{v}=\begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$. [Level:2] [CO2]

4. Give the standard matrix for horizontal and vertical expansion. [Level:2] [CO2]

5. When can we say that a set of vectors is linearly dependent. [Level:1] [CO4]

6. Define a linear transformation. [Level:1] [CO2]

7. Let $A = \begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix}$. Compute $3I_2 - A$ and $(3I_2)A$. [Level:3] [CO3]

8. Given $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, find a vector in Nul A [Level:3] [CO4]

9. State the invertible matrix theorem. [Level:1] [CO5]

10. Show that 7 is an eigenvalue of $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$. [Level:3] [CO5]

(Ceiling: 24 Marks)

Part B (Paragraph questions/Problem)

Answer ***all*** questions. Each question carries 6 marks.

11. Convert the following matrix in to reduced row echelon form. [Level:2] [CO1]

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{bmatrix}$$

12. Let A= $\begin{bmatrix} 1 & 3 & 4 \\ -4 & 2 & -6 \\ -3 & -2 & -7 \end{bmatrix}$ and $\mathbf{b}$= $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. Is the equation $A\mathbf{X} = \mathbf{b}$ consistent for all possible b_1, b_2, b_3? [Level:3] [CO2]

13. Determine by inspection if the given set is linearly dependent, [Level:2] [CO4]

(a) $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix}, \; \mathbf{v}_2 = \begin{bmatrix} 2 \\ 0 \\ 9 \end{bmatrix}, \; \mathbf{v}_3 = \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}, \mathbf{v}_4 = \begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix}.$

(b) $\mathbf{v}_1 = \begin{bmatrix} -2 \\ 4 \\ 6 \\ 10 \end{bmatrix}, \; \mathbf{v}_2 = \begin{bmatrix} 3 \\ -6 \\ -9 \\ 15 \end{bmatrix}.$

14. Let $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}, \mathbf{u} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$, and define transformation T: $R^2 \rightarrow R^3$ by [Level:3] [CO2]

$$T(\mathbf{x}) = A\mathbf{x} = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 3x_2 \\ 3x_1 + 5x_2 \\ -x_1 + 7x_2 \end{bmatrix}.$$

Find an $\mathbf{x}$ in R^2 whose image under T is $\mathbf{b}$.

15. Let A be an invertible $n \times n$ matrix, and let B be an $n \times p$ matrix. Show that the equation $AX = B$ has a unique solution $A^{-1}B$. [Level:3] [CO3]

16. Let $\mathbf{x}$ be a vector in a subspace H with a basis $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$. Find the $\mathcal{B}$-coordinate vector of $\mathbf{x}$. Where $\mathbf{b}_1 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} -2 \\ 7 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} -3 \\ 7 \end{bmatrix}.$ [Level:3] [CO4]

17. Find the characteristic polynomial of $A = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}.$ [Level:3] [CO5]

18. Let $A = \begin{bmatrix} -3 & 12 \\ -2 & 7 \end{bmatrix}, \mathbf{v_1} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\mathbf{v_2} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ be the eigen vectors of A. Use this information to diagonalize A. [Level:3] [CO5]

(Ceiling: 36 Marks)

Part C (Essay questions)

Answer any ***one*** question. The question carries 10 marks.

19. Determine if the following homogeneous system has a nontrivial solution. Then describe the solution set. [Level:3] [CO1]

$$\begin{aligned} 3x_1 + 5x_2 - 4x_3 &= 0 \\ -3x_1 - 2x_2 + 4x_3 &= 0 \\ 6x_1 + x_2 - 8x_3 &= 0 \end{aligned}$$

20. Use the inversion algorithm to find the inverse of the matrix [Level:3] [CO3]

$$A = \begin{bmatrix} 1 & -2 & -1 \\ 1 & 5 & 6 \\ 5 & -4 & 5 \end{bmatrix}$$

(1 × 10 = 10 Marks)

****