24U3122 (Pages: 2) Name : ....

Reg. No : ....

THIRD SEMESTER UG DEGREE EXAMINATION, NOVEMBER 2025

(FYUGP)

CC24UMAT3MN202 - DIFFERENTIAL EQUATIONS AND FOURIER SERIES

(Mathematics - Minor Course)

(2024 Admission - Regular)

Time: 2.0 Hours Maximum: 70 Marks

Credit: 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 3 marks.

1. Check whether the differential equation $\frac{dy}{dx} = \frac{xy^2 - \cos x \sin x}{y(1-x^2)}$ is exact or not. [Level:2] [CO1]
2. Solve $\cos x(e^{2y} - y)\frac{dy}{dx} = e^y \sin 2x$. [Level:3] [CO1]
3. Find an integrating factor for the equation $(10 - 6y + e^{-3x})dx - 2dy = 0$. [Level:2] [CO1]
4. Find the particular solution of $y'' - 10y' + 25y = 30x + 3$ [Level:3] [CO2]
5. Find the half range cosine series of the function [Level:3] [CO3]
$$f(x) = \begin{cases} 1, & 0 < x < \frac{1}{2} \\ 0, & \frac{1}{2} \leq x < 1 \end{cases}$$
6. Define the solution of a linear partial differential equation. [Level:1] [CO3]
7. Determine the polar form of the complex number $z = 6i$. [Level:3] [CO5]
8. By applying the algebra of complex numbers, reduce the expressions (a) $2i^3 - 3i^2 + 5i$ (b) $(2 - 3i)(4 + i)$ in the standard form $a + ib$. [Level:3] [CO2]
9. Show that the function $f(z) = \text{Re}(z)$ is not analytic at any point. [Level:2] [CO5]
10. Evaluate the function $f(z) = 2x - y^2 + i(xy^3 - 2x^2 + 1)$ at the points (a) $2i$ (b) $2 - i$ [Level:3] [CO5]

(Ceiling: 24 Marks)

Part B (Paragraph questions/Problem)

Answer ***all*** questions. Each question carries 6 marks.

11. Solve $(x^2 + y^2)dx + (x^2 - xy)dy = 0$. [Level:3] [CO1]
12. Solve (a) $\frac{dy}{dx} - 3y = 6$ (b) $\frac{dy}{dx} = 5y$. [Level:3] [CO1]

13. Verify that the indicated function is a solution of the given differential equation. [Level:2] [CO1]

(a) $\frac{dy}{dx} = xy^{\frac{1}{2}}$; $y = \frac{1}{16}x^4$ (b) $\frac{dy}{dt} + 20y = 24$; $y = \frac{6}{5} - \frac{6}{5}e^{-20t}$.

14. Verify that the functions x^3, x^4 form a fundamental set of solutions of the differential equation $x^2y'' - 6xy' + 12y = 0$ on the interval $(0, \infty)$. Form the general solution. [Level:2] [CO2]

15. Solve the initial-value problem $y'' + y' + 2y = 0, y(0) = 0, y'(0) = 0$ [Level:3] [CO2]

16. (a) Define the Fourier Sine series of an odd function on the interval $(-p, p)$. [Level:3] [CO3]

(b) Find the Fourier sine seires of $f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 \leq x < \pi \end{cases}$

17. Compute $(2 - 2i)^5$ [Level:3] [CO5]

18. Sketch the set of points in the complex plane satisfying the inequality $\text{Im}(z - i) < 5$. Determine whether the set is an open set, closed set, connected set and domain. [Level:2] [CO5]

(Ceiling: 36 Marks)

Part C (Essay questions)

Answer any ***one*** question. The question carries 10 marks.

19. (a) Find the Fourier series of the function $f(x) = \begin{cases} 1, & -1 < x < 0 \\ x, & 0 \leq x < 1 \end{cases}$ on the given interval. Give the number to which the Fourier series converges at a point of discontinuity of f. [Level:3] [CO3]

(b) Find the Fourier series of the function $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x^2, & 0 \leq x < \pi \end{cases}$ on the given interval. Give the number to which the Fourier series converges at a point of discontinuity of f..

20. Compute all roots of $(-1 + i)^{1/3}$ [Level:3] [CO5]

(1 × 10 = 10 Marks)

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