24U3123 (Pages: 2) Name : .....

Reg. No : .....

THIRD SEMESTER UG DEGREE EXAMINATION, NOVEMBER 2025

(FYUGP)

CC24UMAT3MN203 - MATRIX ALGEBRA AND VECTOR CALCULUS

(Mathematics - Minor Course)

(2024 Admission - Regular)

Time: 2.0 Hours Maximum: 70 Marks

Credit: 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 3 marks.

1. Show that the vectors $\mathbf{a} =< -3, -1, 4 >$ and $\mathbf{b} =< 2, 14, 5 >$ are orthogonal. [Level:2] [CO1]

2. Find $\mathbf{a} \times \mathbf{b}$ for the vectors $\mathbf{a} = \mathbf{i} - \mathbf{j}$ and $\mathbf{b} = 3\mathbf{j} + 5\mathbf{k}$. [Level:2] [CO1]

3. Find a parametric equation for the line passing through the points $(2, -1, 8)$ and $(5, 6, -3)$. [Level:2] [CO1]

4. Find the level curve of $f(x, y) = y - x^2$ passing through the point $(2, 5)$. [Level:2] [CO2]

5. Let $F(x, y, z) = xz\mathbf{i} + yz\mathbf{j} + xy\mathbf{k}$. Find div $\mathbf{F}$. [Level:2] [CO2]

6. Find $r'(t)$ and $r''(t)$ if $r(t) =< te^{2t}, t^3, 4t^2 - t >$. [Level:2] [CO2]

7. Evaluate $\iint_R y\, dA$ where $R = \{0 \leq x \leq 2,\ 0 \leq y \leq 4\}$. [Level:3] [CO3]

8. Evaluate $\iint_S y^2\, dS$, where S is that portion of the cylinder $x = y^2$ in the first octant bounded by $y = 0,\ y = 2,\ z = 0,\ z = 3$. [Level:3] [CO3]

9. Write the augmented matrix for the system: [Level:2] [CO4]

   $3x + 5y = 2z \quad 5x + 4y = 3z - 2 \quad x = z + 6.$

10. Check whether $A = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ is a solution of the system : [Level:3] [CO4]

    $x + 2y - z = 0,\ 2x + y + z = 0,\ x - y + 2z = 0.$

(Ceiling: 24 Marks)

Part B (Paragraph questions/Problem)

Answer ***all*** questions. Each question carries 6 marks.

11. If the distance between the points $(x, x, 1)$ and $(0, 3, 5)$ is 5, then find the value of x. [Level:2] [CO1]

12. Let $\mathbf{a} =< 2, 1 >$ and $\mathbf{b} =< 5, 6 >$. Find a unit vector in the same direction and in the opposite direction of the vector $2\mathbf{a} - 3\mathbf{b}$. [Level:2] [CO1]

13. Find the directional derivative of $F(x, y, z) = 2x - y^2 + z^2$ at $(4, -4, 2)$ in the direction of origin . [Level:2] [CO2]

14. a)If $r = x^2 + y^5 z^3$ and $x = uve^{2s}$, $y = u^2 - v^2 s$, $z = \sin(uvs^2)$, find $\frac{\partial r}{\partial s}$. [Level:2] [CO2]
b) If $z = u^2 v^3 w^4$ and $u = t^2$, $v = 5t$, $w = t^3 + t$, find $\frac{dz}{dt}$.

15. a) Evaluate $\int_C x\, y^2\, dx$ where C is parameterized by $x = t, y = t^2,\ 0 \le t \le 2$. [Level:2] [CO3]
b) Evaluate $\int_C x\, y^2\, dx$ where C is parameterized by
$x = 4\cos t,\ y = 4\sin t,\ 0 \le t \le \frac{\pi}{2}$.

16. Convert $(6, \frac{\pi}{4}, \frac{\pi}{3})$ in spherical coordinates to rectangular coordinates and cylindrical coordinates. [Level:3] [CO3]

17. Find the rank of $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix}$. [Level:2] [CO4]

18. Let $A = \begin{bmatrix} 5 & -1 & 0 \\ 2 & 0 & 1 \\ 4 & 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 & 2 \\ -1 & 1 & 4 \\ 3 & -2 & 1 \end{bmatrix}$. Find $(B^2 - 2A)^T$. [Level:2] [CO4]

(Ceiling: 36 Marks)

Part C (Essay questions)

Answer any ***one*** question. The question carries 10 marks.

19. a) Find an equation of the plane with normal vector $\mathbf{n} =< 2, 8, -5 >$ and containing the point $(4, -1, 3)$. [Level:2] [CO1]
b) Find an equation of the plane containing the points $(1, 0, -1), (3, 1, 4), (2, -2, 0)$.

20. a) Verify $K = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ is an eigen vector of $A = \begin{pmatrix} 0 & -1 & -3 \\ 2 & 3 & 3 \\ -2 & 1 & 1 \end{pmatrix}$. [Level:3] [CO5]
b) Find eigenvalues and eigenvectors of $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

(1 × 10 = 10 Marks)

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