24U301S (Pages: 2) Name: ....

Reg.No: ....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2025

(CBCSS - UG)

CC19UMTS3B03 / CC20UMTS3B03 - CALCULUS OF SINGLE VARIABLE - 2

(Mathematics - Core Course)

(2019 to 2023 Admissions - Supplementary/Improvement)

Time : 2.5 Hours Maximum : 80 Marks

Credit : 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Write the expression $x^{\cos x}$ as an exponent with base e.
2. Evaluate $(a)\cos\left(\sin^{-1}\frac{1}{\sqrt{2}}\right)$ $\quad (b)\tan\left(\sin^{-1}\frac{\sqrt{2}}{2}\right)$.
3. Evaluate $\int \frac{\sinh x}{1+\cosh x}dx$.
4. Evaluate $\lim\limits_{\theta\to 0}\frac{\tan 2\theta}{\theta}$
5. Define geometric series.
6. Determine whether the series $\sum_{n=1}^{\infty} n^{-1.001}$ converges or diverges.
7. Determine whether the series $\sum_{n=1}^{\infty}(-1)^n \frac{2n}{4n-1}$ converges or diverges.
8. Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$ is absolutely convergent.
9. Find $\frac{dy}{dx}$, if $x=\sqrt{t^2+1}$, $y=tlnt$.
10. The point $\left(-\sqrt{3},-\sqrt{3}\right)$ is given in rectangular coordinates. Find its representation in polar coordinates.
11. Find the area of the region bounded by the curve $r=\sqrt{cos\theta}$ and the rays $\theta=0$ and $\theta=\frac{\pi}{2}$.
12. Sketch the cylinder $9x^2+4y^2=36$.
13. The point $\left(3,\frac{\pi}{3},\frac{\pi}{4}\right)$ is expressed in spherical coordinates. Find its rectangular coordinates.
14. Find $\overline{\gamma}'(0)$ of $\overline{\gamma}(t)=\left\langle e^t,e^{-2t}\right\rangle$
15. Define smooth curve.

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Find $\int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx$.

17. Find the derivative of $f(x) = x^2 \log(e^{2x} + 1)$.

18. Evaluate $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$ and interpret your result geometrically.

19. Determine whether the sequence $a_n = 3 - \frac{1}{n}$ is monotonic. Is the sequence bounded?

20. Find the radius of convergence and the interval of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n n!(x-1)^n}{1.3.5.\ldots.(2n-1)}$.

21. A proposed training course for a yacht is represented by the parametric equations $x = sint$ and $y = sin2t, 0 \leq t \leq 2\pi$ where x and y are measured in miles.
    (i) Show that the rectangular equation of the course is $4x^4 - 4x^2 + y^2 = 0$.
    (ii) Describe the course.

22. Find the velocity and position vector of an object with acceleration $\overline{a}(t) = \overline{i} - t\overline{j} + (1+t)\overline{k}$ with $\overline{v}(0) = \overline{i} + \overline{k}, \overline{\gamma}(0) = \overline{j} + \overline{k}$.

23. Find $\overline{T}(t)$ and $\overline{N}(t)$ if the curve C is defined by $\overline{\gamma}(t) = \langle e^t \cos t, e^t \sin t, e^t \rangle$

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. Show that the function $y = 2x^2 + 3x^2 \ln x$ is a solution of the differential equation $x^2y'' - 3xy' + 4y = 0$

25. a) Show that the series $\sum_{n=1}^{\infty} \frac{2n^2+n}{\sqrt{4n^7+3}}$ is divergent.
    b) Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{2n^2+1}$ is convergent or not.

26. Find $\int e^{-x^2} dx$.

27. $i)$ Find an equation of the plane passing through the points $(-1, 3, 0)$ and $(2, -1, 4)$ and perpendicular to the plane $3x - 4y + 5z = 1$.
    $ii)$ Find the parametric equations for the line of intersection of the planes defined by $3x - y + 2z = 1$ and $2x + 3y - z = 4$.

(2 × 10 = 20 Marks)

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