24U302S (Pages: 2) Name: ....

Reg.No: ....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2025

(CBCSS - UG)

CC19UMTS3C03 / CC20UMTS3C03 - MATHEMATICS - 3

(Mathematics - Complementary Course)

(2019 to 2023 Admissions - Supplementary/Improvement)

Time : 2.00 Hours Maximum : 60 Marks

Credit : 3

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Prove $\nabla(cf) = c\nabla f$ where c is a constant and f is a differentiable function of two variables.
2. Define curl and divergence of a vectorfield.
3. Evaluate $\int_c xy^2 ds$ on the quarter circle C defined by $x = 4\cos t, \quad y = 4\sin t, \quad 0 \leq t \leq \frac{\pi}{2}$
4. Convert the equation $r = 5\sec\theta$ to rectangular coordinates.
5. What do you meant by a *one-to-one* transformation?
6. Express $1 + i$ in polar form.
7. Find the value of $\ln(-1 - i)$
8. Evaluate $\int_C \overline{z} dz$, where C is the curve given by $x = 3t, y = t^2, \ -1 \leq t \leq 4$.
9. Define continuity at a point.
10. Show that $f(z) = y + ix$ is not analytic at any point
11. Calculate circulation around and net flux across the circle: $|z| = 1$ for $f(z) = (1 + i)z$
12. Evaluate $\oint_C \frac{1+2e^z}{z} dz$ where C is $|z| = 1$

(Ceiling: 20 Marks)

Part B (Short essay questions - Paragraph)

Answer ***all*** questions. Each question carries 5 marks.

13. Find the length of the curve $\mathbf{r}(t) = t\mathbf{i} + t\cos t\mathbf{j} + t\sin t\mathbf{k}$ on the interval $0 \leq t \leq \pi$.
14. Using Green's theorem evaluate the line integral $\oint_C (2ydx + 5ydy)$, where C is the circle $(x-1)^2 + (y+3)^2 = 25$ taken in anticlockwise direction.

15. If $\mathbf{F} = xy\mathbf{i} + y^2z\mathbf{j} + z^3\mathbf{k}$, evaluate $\iint_S \mathbf{F}.\mathbf{n}dS$ where S is the unit cube defined by $0 \le x \le 1, \quad 0 \le y \le 1, \quad 0 \le z \le 1.$

16. Verify that the function $u(x, y) = x^2 - y^2$ is harmonic. Find v, the harmonic conjugate of u. Also form the corresponding analytic function $f(z)$.

17. State Cauchy's integral formula. Using Cauchy's integral formula evaluate $\oint_C \frac{e^z}{z - \pi i} dz$ where C is $|z| = 4$.

18. Express $sinh(\frac{3\pi}{2})i$ in the form $a + ib$.

19. Evaluate $\oint_C \left(z^3 + z^2 + Re(z)\right) dz$, where C is the triangle with vertices $z = 0, z = 1, z = 1 + 2i$

(Ceiling: 30 Marks)

Part C (Essay questions)

Answer any ***one*** question. The question carries 10 marks.

20. Show that the line integral $\int_{(1,1,1)}^{(2,4,8)} yzdx + xzdy + xydz$ is path independent. By finding a potential function evaluate the integral.

21. Evaluate $\int_{\pi}^{i} e^z \cos zdz$
Evaluate $\int_{i}^{i+1} ze^z dz$

(1 × 10 = 10 Marks)

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