23U501 (Pages: 2) Name: ……..……….……..

Reg. No: ………………….

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2025

(CBCSS-UG)

(Regular/Supplementary/Improvement)

CC20UMTS5B05 - ABSTRACT ALGEBRA

(Mathematics – Core Course)

(2020 Admission onwards)

Time: 2 ½ Hours Maximum: 80 Marks

Credit: 4

Section-A

Answer *all* questions. Each question carries 2 marks.

1. Find the multiplicative inverse of $[3]_4$ in $\mathbb{Z}_4$.
2. Check whether the relation ~ on $\mathbb{R}$ defined by a ~ b if a - b ∈ $\mathbb{Q}$ is a equivalence relation.
3. Compute the product $\sigma\tau$, where $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$.
4. Define identity element in a group G and find the identity element in the group of non zero real numbers under multiplication.
5. Give an example of an infinite abelian group.
6. Find the order of $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ in $GL_2(\mathbb{R})$.
7. Is $\mathbb{Z}_2 \times \mathbb{Z}_3$ Cyclic ? Justify your answer.
8. Define transposition. Express (1,2,3) (4,5) as a product of transpositions.
9. Find the order of the permutation (1 , 2, 5) (2, 3 , 4) (5 , 6).
10. Show that the function $\varphi: \mathbb{Z} \to m\mathbb{Z}$ defined by $\varphi(x) = mx$ is a homomorphism.
11. Find any two cosets of $4\mathbb{Z}_{12}$ in $\mathbb{Z}_{12}$.
12. Show that all subgroups of index 2 are normal.
13. Define a simple group and give an example of a simple cyclic group.
14. Is $\mathbb{Z}[x]$ an integral domain? Justify your answer.
15. Define center of a group G.

(Ceiling: 25 Marks)

Section - B

Answer *all* questions. Each question carries 5 marks.

16. Prove that if $(a, n) = 1$, then $a^{\varphi(n)} \equiv 1\ (mod\ n)$.
17. Define an equivalence relation on $\mathbb{R}$ that partitions the real line into subsets of length 1.
18. Write the multiplication table of $\mathbb{Z}_8^{\times}$, the group of units modulo 8.
19. Prove that a nonempty subset H of a group G is a subgroup of G if $ab^{-1} \in H$ for all $a, b \in H$.
20. Find two non isomorphic groups of order 4 and explain their distinguishing characteristics.
21. Prove that the homomorphism $\phi: G_1 \to G_2$ is one-to-one if and only if Ker$(\phi) = \{e\}$.
22. State and prove fundamental homomorphism theorem.
23. Prove that any finite integral domain is a field.

(Ceiling: 35 Marks)

Section - C (Essay Type)

Answer any *two* questions. Each question carries 10 marks.

24. a) Define disjoint cycles. Give two disjoint cycles in S_6.

    b) Prove that every permutation in S_n can be written as a product of disjoint cycles.
25. a) State and prove theorem of Lagrange.

    b) Prove that order of an element divides order of the group.
26. a) Prove that every subgroup of a cyclic group is cyclic.

    b) Find all subgroups of $\mathbb{Z}$ under addition.
27. a) State and prove the second isomorphism theorem.

    b) Show that $\mathbb{Z}_n/m\mathbb{Z}_n \cong \mathbb{Z}_m$ if m divides n.

(2 × 10 = 20 Marks)

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