23U502 (Pages: 2) Name: ....

Reg. No: ....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2025

(CBCSS-UG)

(Regular/Supplementary/Improvement)

CC20UMTS5B06 - BASIC ANALYSIS

(Mathematics – Core Course)

(2020 Admission onwards)

Time: 2 ½ Hours Maximum: 80 Marks

Credit: 4

Section A

Answer ***all*** questions. Each question carries 2 marks.

1. If $a, b \in \mathbb{R}$ with $ab = 0$, prove that either $a = 0$ or $b = 0$.
2. Let $a, b, c \in \mathbb{R}$. If $a > b$ and $b > c$, prove that $a > c$.
3. Define denumerable set. Give an example.
4. Find all real numbers x satisfying the inequality $x^2 > 3x + 4$.
5. State Cantor's theorem.
6. Prove that $\lim_{n\to\infty} \frac{\sin n}{n} = 0$.
7. State the Nested Interval Property.
8. Show that the sequence $((-1)^n)$ is divergent.
9. Find the limit superior and limit inferior of the sequence $(1, 2, 3, 1, 2, 3, ...)$.
10. Prove that every convergent sequence of real numbers is bounded.
11. Prove that the union of any collection of open sets in $\mathbb{R}$ is open.
12. Find the reciprocal of $z = 2 - 3i$.
13. Find the modulus of the complex number $z = -9i$.

14. Find the real and imaginary parts of the complex function $f(z) = z^2$ in polar coordinates.

15. Find the image of the line segment from 1 to i under the complex mapping $\omega = iz$.

(Ceiling: 25 Marks)

Section B

Answer ***all*** questions. Each question carries 5 marks.

16. State and prove Bernoulli's Inequality.

17. Prove that $\sup(A + B) = \sup A + \sup B$, for any bounded nonempty subsets of $\mathbb{R}$.

18. State and Prove the density theorem.

19. Prove that the set $\mathbb{R}$ of real numbers is not countable.

20. Show that every contractive sequence is a Cauchy sequence and therefore is convergent.

21. Prove that a subset of $\mathbb{R}$ is closed if and only if it contains all of its cluster points.

22. Pove that $\sqrt{2}|z| \geq |Rez| + |Imz|$.

23. Find the image of half plane $Im(z) \leq 1$ under the complex mapping $w = iz + 2$.

(Ceiling: 35 Marks)

Section C

Answer any ***two*** questions. Each question carries 10 marks.

24. (i) Let S be a nonempty bounded set in $\mathbb{R}$, for any $a \in \mathbb{R}$ and let $aS = \{as : s \in S\}$. Prove that $sup(aS) = a\,sup\ S$, if $a > 0$ and $sup(aS) = a\,inf S$, if $a < 0$.

    (ii) State and prove the Archimedean property.

25. State and prove Monotone convergence theorem.

26. State and prove Cauchy's convergence criterion.

27. (i) Sketch the region: $Rez \geq 1$ and $Imz \leq 1$.

    (ii) Find the four fourth roots of $z = 1 + i$.

(2 × 10 = 20 Marks)

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