23U505 (Pages: 2) Name: ....

Reg. No: ....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION- NOVEMBER 2025

(CBCSS-UG)

(Regular/Supplementary/Improvement)

CC20UMTS5B09 - INTRODUCTION TO GEOMETRY AND THEORY OF EQUATIONS

(Mathematics - Core Course)

(2020 Admission onwards)

Time: 2 Hours

Maximum: 60 Marks

Credit : 3

Part A

Answer *all* questions. Each question carries 2 marks.

1. Find the focus, vertex, axis and directrix of the parabola E with equation $y^2 = 2x$ and parametric equations $x = \frac{1}{2}t^2, y = t(t \in R)$.
2. Determine the slope of the tangent to the curve in R^2 with parametric equations $x = 2\cos t + \cos 2t + 1, y = 2\sin t + \sin 2t$ at the point with parameter t, where t is not a multiple of π.
3. Determine the inverse of the Euclidean transformation given by $t(x) = \begin{pmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{pmatrix} x + \begin{pmatrix} 1 \\ -2 \end{pmatrix}$.
4. Explain with an example that an affine transformation is not necessarily a parallel projection.
5. Without actual division show that $x^4 + 3x^3 + 3x^2 + 3x + 2$ is divisible by $x + 2$.
6. By synthetic division, find the quotient and remainder in the division of $2x^4 - 6x^3 + 7x^2 - 5x + 1$ by $x + 2$.
7. Write the cubic equation with the roots $1, 1 + i, 1 - i$.
8. Find an upper limit of positive roots of the equation $2x^5 - 7x^4 - 5x^3 + 6x^2 + 3x - 10 = 0$.
9. State Rolle's Theorem.
10. If two polynomials $f1$ and $f2$ are divisible by g, then show that for arbitrary polynomials l and $l1$ the polynomial $lf + l1f1$ will be divisible by g.
11. Verify that the equation $x^3 - 7x + 7 = 0$ have roots in the intervals $(-4, -3)$, $(1, \frac{3}{2})$, $(\frac{3}{2}, 2)$.
12. Separate the roots of the equation $f(x) = 2x^5 - 5x^4 + 10x^2 - 10x + 1$.

(Ceiling: 20 Marks)

Part B

Answer *all* questions. Each question carries 5 marks.

13. State and prove Focal distance property of Hyperbola.
14. Define the affine transformation which maps the points $(0, 0)$, $(1, 0)$,and $(0, 1)$ to the points $(3, 2)$, $(5, 8)$ and $(7, 3)$ respectively.
15. Prove that a parallel projection preserves ratios of lengths along a given straight line.
16. Examine whether the equation $x^3 - 2x^2 - 25x + 50 = 0$ has integral roots or not.
17. By the method of detached coefficients find the quotient and the remainder when $2x^7 - 3x^6 + x^5 - 3x^4 + 5x^3 - 4x^2 + 2x - 1$ is divided by $2x^3 - 3x^2 + x - 1$
18. Solve the biquadratic equation $x^4 + 4x - 1 = 0$.
19. Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$ if the product of two roots is 1.

(Ceiling: 30 Marks)

Part C

Answer any *one* question. The question carries 10 marks.

20. Prove that the conic E with equation $3x^2 - 10xy + 3y^2 + 14x - 2y + 3 = 0$ is a hyperbola. Determine its centre, and its major and minor axes.

21. a. Solve the cubic equation $x^3 - 6x - 6 = 0$ using Cardan's formula.

    b. Prove that $\sqrt[3]{\sqrt{5}+2} - \sqrt[3]{\sqrt{5}-2} = 1$.

(1 × 10 = 10 Marks)

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