

SECOND SEMESTER B.Voc. DEGREE EXAMINATION, APRIL 2026

(FYUGP)

CC25STA2MN113 - PROBABILITY THEORY

(Statistics - Minor Course)

(2025 Admission - Regular)

Time: 2.0 Hours

Maximum: 70 Marks

Credit: 4

Part A (Short answer questions)Answer **all** questions. Each question carries 3 marks.

1. Distinguish between independent and dependent events. [Level:2] [CO1]
2. A coin is tossed three times. Write the sample space of the experiment. [Level:1] [CO1]
3. In a class, 40% of the students like Mathematics and 25% of students like Physics and 15% like both the subjects. One student select at random, find the probability that he likes Physics if it is known that he likes Mathematics. [Level:3] [CO2]
4. It is 8:5 against the wife who is 40 years old living till she is 70 and 4:3 against her husband now 50 living till he is 80. Find the probability that both will be alive. [Level:3] [CO2]
5. What is the probability that a leap year selected at random will contain 53 Sundays? [Level:3] [CO1]
6. For a discrete random variable X with probability distribution: [Level:3] [CO3]

X	-2	-1	0	1	2
P(X=x)	0.1	0.2	0.3	k	0.2

Find the value of: a) k b) $P(-1 \leq x \leq 1)$

7. Define skewness and kurtosis in terms of moments. [Level:1] [CO4]
 8. What is the relationship between the characteristic functions of X and aX+b+? [Level:2] [CO4]
 9. If the first raw moment is 5 and the second raw moment is 29, find the variance. [Level:3] [CO4]
 10. The joint pdf's of two r.v.s. X and Y is given by [Level:3] [CO5]
- $$f(x, y) = \begin{cases} 24x(1 - y), & 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases}$$
- Find the marginal pdf's of X and Y.

(Ceiling: 24 Marks)

Part B (Paragraph questions/Problem)

Answer **all** questions. Each question carries 6 marks.

11. Check whether the following is a density function or not [Level:2] [CO3]
- $$f(x) = \begin{cases} 0, & x \leq a \\ \frac{1}{a^2}(a+x), & -a < x \leq 0 \\ \frac{1}{a^2}(a-x), & 0 \leq x < a \\ 0, & \text{otherwise} \end{cases}$$
12. If X is a continuous random variable with pdf $f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$. Find the pdf of $Y = 3X + 1$ [Level:3] [CO3]
13. A r.v X has the probability density function $f(x) = \begin{cases} k\frac{1}{1+x^2}, & -\infty < x < \infty \\ 0, & \text{otherwise} \end{cases}$. Determine k and the distribution function [Level:3] [CO3]
14. If $f(x) = \begin{cases} \frac{1}{2}, & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$. Find the expectation of x [Level:3] [CO4]
15. Find the mgf for $f(x) = \frac{1}{8}(1+x), 2 < x < 4$ [Level:3] [CO4]
16. Given the joint p.d.f of two random variables X and Y as [Level:3] [CO5]
- $$f(x, y) = 3x^2y + 3y^2x, 0 \leq x \leq 1; 0 \leq y \leq 1.$$
- Find $(\frac{1}{2} \leq X \leq \frac{3}{4} / \frac{1}{3} \leq Y \leq \frac{2}{3})$
17. Two fair dice are tossed simultaneously. Let X denote the sum of the numbers shown by the two dice and Y denote the absolute value of their differences. Obtain the joint probability masses of (X, Y) . Also find the probabilities of: [Level:3] [CO5]
- a) $(X = 8 \cap Y \geq 3)$ b) $((X < Y)$
18. Given the joint probability masses of (X, Y) as, [Level:4] [CO5]
- $$f(x, y) = \frac{1}{4}; \text{ for } (x, y) = (1, 1), (1, 2), (2, 1), (2, 2).$$
- Examine whether X and Y are independent

(Ceiling: 36 Marks)

Part C (Essay questions)

Answer any **one** question. The question carries 10 marks.

19. It is estimated that 50% of emails are spam emails. Some software has been applied to filter these spam emails before they reach your inbox. A certain brand of software claims that it can detect 99% of spam emails, and the probability for a false positive (a non-spam email detected as spam) is 5%. Now if an email is detected as spam, then what is the probability that it is in fact a non-spam email? [Level:3] [CO2]

20. Let X and Y have joint probability density function

[Level:3] [CO5]

$$f(x, y) = \begin{cases} 1, & 0 < y < 2x < 2 \\ 0, & \text{otherwise} \end{cases}. \text{ Find } E(XY^2)$$

(1 × 10 = 10 Marks)
