25P201 (Pages: 2) Name: ....

Reg.No: ....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2026

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19PMTH2C06 - ALGEBRA - II

(Mathematics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part A

Answer any ***all*** questions. Each question carries 1 weightage.

1. Prove that F is a field, every proper non trivial prime ideal of $F[x]$ is maximal.
2. Prove that $\mathbb{R}(i) \cong \mathbb{C}$
3. Show that regular pentagon is constructible.
4. Let E be a finite extension of degree n over a finite field F. Prove that if F has q elements then E has q^n elements.
5. Prove that any two algebraic closures of a field F are isomorphic.
6. Prove that any field F is a splitting field over F.
7. State Primitive Element Theorem.
8. Prove that the polynomial $x^n - a$ is solvable by radicals over $\mathbb{Q}$.

(8 × 1 = 8 Weightage)

Part B

Answer any ***two*** questions each unit. Each question carries 2 weightage.

UNIT - I

9. A finite extension field E of a field F is an algebraic extension of F . What about the converse?
10. Find a basis for $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ over $\mathbb{Q}$.
11. Let E be an extension field of F, then prove that $\overline{F}_E = \{\alpha \in E : \alpha \text{ is algebraic over } F\}$ is a subfield of E.

UNIT - II

12. If F a field of prime characteristic p, then $(\alpha+\beta)^{p^n} = \alpha^{p^n} + \beta^{p^n}$ for all $\alpha, \beta \in F$ and all positive integers n.

13. If E is a finite extension of F, then $\{E : F\}$ divides $[E : F]$.

14. If E is a finite extension of F, then prove that E is separable over F if and only if each α in E is separable over F.

UNIT - III

15. Describe the Galois group $G(\mathbb{Q}(\sqrt{2})/\mathbb{Q})$.

16. Find the splitting field of $x^4 + 1$ over $\mathbb{Q}$.

17. Find $\Phi_8(x)$ over $\mathbb{Z}_2$.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Let R be a commutative ring with unity and M. is an ideal in R .Then prove that M is a maximal ideal of R if and only if R/M is a field.

19. State and Prove Kroneckers Theorem.

20. State and Prove The Conjugation Isomorphism theorem.

21. Prove that the Galois group of pth cyclotomic extension of $\mathbb{Q}$ for a prime p is cyclic of order $p - 1$.

(2 × 5 = 10 Weightage)

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