25P202 (Pages: 2) Name: ....

Reg.No: ....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2026

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19PMTH2C07 - REAL ANALYSIS - II

(Mathematics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Prove that for any set A and number y, $m^*(A+y) = m^*(A)$.
2. Explicitly find a choice set for the equivalence relation on a set of all rational numbers.
3. State Simple Approximation lemma.
4. Define Lebesgue integral.
5. State Lebesgue Dominated Convergence Theorem.
6. Define Uniform Integrability of measurable functions.
7. State Vitali Covering Lemma.
8. Let f be a Lipschitz function on $[a, b]$. Show that $TV(f) \leq c(b-a)$

(8 × 1 = 8 Weightage)

Part B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT - I

9. If $\{A_k\}_{k=1}^{\infty}$ is an ascending collection of measurable sets, then prove that $m(\bigcup_{k=1}^{\infty} A_k) = \lim_{k\to\infty} m(A_k)$.
10. Let f and g be measurable functions on E that are finite a.e. on E. Prove that for any α and β, $\alpha f + \beta g$ and fg is measurable on E.
11. Assume E has finite measure. Let $\{f_n\}$ be a sequence of measurable functions on E that converges pointwise on E to the real-valued function f. Then prove thar for each $\epsilon > 0$, there is a closed set F contained in E for which $\{f_n\} \to f$ uniformly on F and $m(E \setminus F) < \epsilon$.

UNIT - II

12. Prove that for any simple functions φ and ψ defined on a set of finite measure E. Prove that for any α and β, $\int_E(\alpha\varphi+\beta\psi)=\alpha\int_E\varphi+\beta\int_E\psi$. Moreover, $\varphi\leq\psi$ on E, then $\int_E\varphi\leq\int_E\psi$.
13. Prove that linearity and monotonicity of integration holds for a bounded measurable functions.
14. State and prove Chebychev's Inequality.

UNIT - III

15. A function f is of bounded variation on the closed, bounded interval $[a,b]$ if and only if it is the difference of two increasing functions on $[a,b]$.
16. Prove that a function on a closed, bounded interval $[a,b]$ is absolutely continuous on $[a,b]$ if and only if it is an indefinite integral over $[a,b]$.
17. State and prove Minkowski's Inequality.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Prove that outer measure of an interval is its length.
19. Explain the construction of the Cantor set and prove that the Cantor set is a closed measurable and uncountable set of measure 0.
20. Let f be a bounded function on the closed,bounded interval $[a,b]$. Then prove that f is Riemann integrable over $[a,b]$ if and only if the set of points in $[a,b]$ at which f fails to be continuous has measure zero.
21. Prove that $L^p(E)$ is a normed linear space for $1\leq p\leq\infty$.

(2 × 5 = 10 Weightage)

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