25P203 (Pages: 2) Name: ....

Reg.No: ....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2026

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19PMTH2C08 - TOPOLOGY

(Mathematics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part A

Answer any ***all*** questions. Each question carries 1 weightage.

1. Let $\mathcal{B}$ be a base for a topology $\mathcal{T}$ on a set X and let $Y \subset X$. Let $\mathcal{B}/Y = \{B \cap Y : B \in \mathcal{B}\}$. Then prove that $\mathcal{B}/Y$ is a base for the toplogy $\mathcal{T}/Y$ on Y .
2. Let $(X, \mathcal{T})$ be a topological space and $A \subset X$. Define the interior of A and neighbourhood of a point $x_0 \in X$.
3. Define projection functions. Prove that projection functions are not closed.
4. Define weak topology determined by the family of functions $\{f_i : i \in I\}$.
5. Show that every closed surjective map is a quotient map.
6. Prove that a space X is locally connected at a point $x \in X$, if and only if for every nieghbourhood N of x the component of N containing x is a a nieghbourhood of x.
7. Prove that a compact subset in a Hausdorff space is closed.
8. State Urysohn's lemma

(8 × 1 = 8 Weightage)

Part B

Answer any ***two*** questions each unit. Each question carries 2 weightage.

UNIT - I

9. Let $f : X \to Y$ be a function where X, Y are metric spaces and let $x_0 \in X$. Then prove that f is continuous at x_0 iff for every open set V in Y containing $f(x_0)$, there exists an open set U in X containing x_0 such that $f(U) \subset V$.
10. Let X be a set and let $\mathcal{T}$ consists of all those subsets of X whose complements are countable together with the empty set. Show that $\mathcal{T}$ is a topology.

11. Define dense subset with an example. Prove that a subset A of a space X is dense in X iff for every non-empty open subset B of X, $A \cap B \neq \phi$

UNIT - II

12. Define first countable space. Prove that every second countable space is first countable.

13. Define disconnected space. Let $f : X \to Y$ be a continuous surjection. Then prove that if X is connected so is Y.

14. Prove that every quotient space of a locally connected space is locally connected.

UNIT - III

15. Differentiate between T_0 space and T_1 spaces with examples.

16. Prove that all metric spaces are T_4.

17. Prove that every regular Lindeloff space is normal.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. Prove that the product topology on $\mathcal{R}^n$ coincides with the usual topology on it.

19. (a) Prove that every continuous real valued function on a compact space is bounded and attains its extrema.
    (a) State and prove Lebesque covering lemma.

20. (a) Let $\mathcal{C}$ be the collection of connected subsets of a space X, such that no two members of $\mathcal{C}$ are mutually separated. Prove that $\cup_{c \in \mathcal{C}} C$ is connected.
    (b) Prove that product of two connected topological spaces is connected.

21. Let A be a closed subset of a normal space X and suppose $f : A \to (-1, 1)$ is a continuous function. Then prove that there exists a continuous function $F : X \to (-1, 1)$ such that $F(x) = f(x)$ for all $x \in A$.

(2 × 5 = 10 Weightage)

****