25P204 (Pages: 2) Name: ....

Reg.No: ....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2026

(CBCSS - PG)

(Regular/Supplementary/Improvement)

CC19PMTH2C09 - ODE AND CALCULUS OF VARIATIONS

(Mathematics)

(2019 Admission onwards)

Time : 3 Hours Maximum : 30 Weightage

Part A

Answer any ***all*** questions. Each question carries 1 weightage.

1. Find a power series solution of the differential equation $(1+x)y' = py$.
2. Determine the nature of the point $x=0$ for the differential equation $x^2y'' + (\sin x)y = 0$.
3. Determine the nature of the point $x=\infty$ for the Euler's equation $y'' + \frac{4}{x}y' + \frac{2}{x^2}y = 0.$
4. Find the critical points of $\dfrac{dx}{dt} = y^2 - 5x + 6,\ \ \dfrac{dy}{dt} = x - y.$
5. Determine the nature and stability properties of the critical point $(0,0)$ of the linear autonomous system $\dfrac{dx}{dt} = 4x - 2y,\ \ \dfrac{dy}{dt} = 5x + 2y$.
6. Verify that (0, 0) is a simple critical point for the system $\dfrac{dx}{dt} = x + y - 2xy,\ \ \dfrac{dy}{dt} = -2x + y + 3y^2.$
7. Using Picard's method of successive approximation, solve the initial value problem $y' = y,\ y(0) = 1$ (start with $y_0(x) = 1$).
8. Show that $f(x,y) = xy$ does not satisfy a Lipschitz condition on the entire plane.

(8 × 1 = 8 Weightage)

Part B

Answer any ***two*** questions each unit. Each question carries 2 weightage.

UNIT - I

9. Find the general solution of $y'' + xy' + y = 0$ in terms of power series in x .
10. Find the general solution of the differential equation $(1-e^x)y'' + \frac{1}{2}y' + e^x y = 0$ near the singular point $x=0$.
11. For the Legendre polynomial $P_n(x)$, prove that $P_n(1) = 1$ and $P_n(-1) = (-1)^n$.

UNIT - II

12. Show that $\begin{cases} x = e^{4t} \\ y = e^{4t} \end{cases}$ and $\begin{cases} x = e^{-2t} \\ y = -e^{-2t} \end{cases}$ are independent solutions of the homogeneous system $\begin{cases} \dfrac{dx}{dt} = x + 3y \\ \dfrac{dy}{dt} = 3x + y \end{cases}$ on every closed interval and then write the general solutions of the system. Also find the particular solution of this system for which $x(0) = 5$ and $y(0) = 1$.

13. Find the general solution of the system $\dfrac{dx}{dt} = x + y,\ \dfrac{dy}{dt} = 4x - 2y.$

14. Show that $(0, 0)$ is an asymptotically stable critical point for the system $\dfrac{dx}{dt} = -2x + xy^3,\ \dfrac{dy}{dt} = -x^2y^2 - y^3.$

UNIT - III

15. If $q(x) < 0$ and $u(x)$ is a n ontrivial solution of $u'' + q(x)u = 0$, then prove that $u(x)$ has atmost one zero.

16. Show that the eigen functions of the boundary value problem $\dfrac{d}{dx}\left[p(x)\dfrac{dy}{dx}\right] + \lambda q(x)y = 0; y(a) = y(b) = 0$ satisfy the relation $\int_a^b q\, y_m(x)\, y_n(x)dx = 0$ if $m \neq n.$

17. Find the plane curve of fixed perimeter and maximum area.

(6 × 2 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

18. State and prove the orthogonality property of Legendre polynomials.

19. (i) Derive Bessel's function of first kind.
(ii) Show that between any two positive zeros of $J_0(x)$, there is a zero of $J_1(x)$.

20. (i) State and prove the orthogonality property of Bessel functions.
(ii) Prove that $\dfrac{d}{dx}[x^p J_p(x)] = x^p J_{p-1}(x)$

21. Derive Euler's equation for an extremal and find the stationary function of $\int_{x_1}^{x_2} [xy' - (y')^2]dx$ which is determined by the boundary conditions $y(0) = 0$ and $y(4) = 3$.

(2 × 5 = 10 Weightage)

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