25U203 (Pages: 2) Name : ....

Reg. No : ....

SECOND SEMESTER UG DEGREE EXAMINATION, APRIL 2026

(FYUGP)

(Regular/Supplementary/Improvement)

CC24UMAT2MN105 - VECTOR SPACES AND LINEAR TRANSFORMATIONS

(Mathematics - Minor Course)

(2024 Admission onwards)

Time: 2.0 Hours Maximum: 70 Marks

Credit: 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 3 marks.

1. Define Zero Vector Space. [Level:1] [CO1]

2. Give the two conditions for a subset W of a vector space V to be a subspace. [Level:1] [CO1]

3. Give a relation between the dimension of a vector space and dimensions of its subspaces. Also give the condition when the dimension of the vector space and its subspace become equal. [Level:2] [CO2]

4. Define the coordinate vector of v relative to the basis S. [Level:1] [CO2]

5. What do you mean by a finite dimensional and infinite dimensional vector space? Give an example for each. [Level:1] [CO2]

6. Write standard matrices for projection onto the x-axis and the y-axis in R^2. [Level:1] [CO3]

7. Use matrix multiplication to find the expansion of $(-1, 2)$ in the x-direction with factor $k = 3$. [Level:2] [CO3]

8. Confirm by multiplication that x is an eigenvector of A, and find the corresponding eigenvalue, where [Level:2] [CO4]
$A = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}$ and $x = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

9. Find $det(A)$ given that $p(\lambda) = \lambda^3 - 5\lambda^2 + 8\lambda - 4$ as its characteristic polynomial. [Level:3] [CO4]

10. Find the eigenvalues of the matrix A^5 where [Level:2] [CO4]
A= $\begin{bmatrix} 5 & 0 & 0 \\ 1 & 4 & 0 \\ -3 & 5 & 3 \end{bmatrix}$

(Ceiling: 24 Marks)

Part B (Paragraph questions/Problem)

Answer ***all*** questions. Each question carries 6 marks.

11. Check whether the set of all invertible $n \times n$ matrices is a subspace of M_{nn}. [Level:3] [CO1]

12. Determine whether the vectors $v_1 = (2, -1, 3)$, $v_2 = (4, 1, 2)$ and $v_3 = (8, -1, 8)$ span the vector space R^3. [Level:3] [CO1]

13. Use Wronskian to show that $f_1(x) = e^x$, $f_2(x) = xe^x$ and $f_3(x) = x^2e^x$ are linearly independent vectors in $C^\infty(-\infty, \infty)$. [Level:2] [CO2]

14. Determine whether the polynomials $P_1 = 1 - x$, $P_2 = 5 + 3x - 2x^2$, $P_3 = 1 + 3x - x^2$ are linearly dependent or linearly independent in P_2. [Level:3] [CO2]

15. Find an equation for the image of the line $y = -4x + 3$ under multiplication by the matrix $A = \begin{bmatrix} 4 & -3 \\ 3 & -2 \end{bmatrix}$. [Level:3] [CO3]

16. Find the standard matrix for a reflection about the xy-plane, followed by a reflection about xz-plane, followed by an orthogonal projection onto the yz-plane in R^3. [Level:2] [CO3]

17. Find the characteristic equation, the eigenvalues, and bases for the eigenspaces of the matrix $A = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$ [Level:3] [CO4]

18. Find a matrix P that diagonalizes [Level:3] [CO4]

A= $\begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

(Ceiling: 36 Marks)

Part C (Essay questions)

Answer any ***one*** question. The question carries 10 marks.

19. Find a basis for and the dimension of the solution space of the homogeneous system [Level:3] [CO2]

$$\begin{aligned} x_1 + 3x_2 - 2x_3 + 2x_5 &= 0 \\ 2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 &= 0 \\ 5x_3 + 10x_4 + 15x_6 &= 0 \\ 2x_1 + 6x_2 + 8x_4 + 4x_5 + 18x_6 &= 0 \end{aligned}$$

20. Show that the operator T:$R^2 \rightarrow R^2$ defined by the equations [Level:3] [CO3]

$$\begin{aligned} w_1 &= x_1 + 2x_2 \\ w_2 &= -x_1 + x_2 \end{aligned}$$

is one-to-one , and find $T^{-1}(w_1, w_2)$.

(1 × 10 = 10 Marks)

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