25U202S (Pages: 2) Name: ....

Reg.No: ....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2026

(CBCSS - UG)

CC19UMTS2C02 / CC20UMTS2C02 - MATHEMATICS - II

(Mathematics - Complementary Course)

(2019 to 2023 Admissions - Supplementary)

Time : 2.00 Hours Maximum : 60 Marks

Credit : 3

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Convert $(4, -\pi)$ from polar coordinates to Cartesian coordinates.
2. Show that $f(x) = \frac{1}{3}x^3 - x$ is not invertible on any open interval containing 1.
3. Sketch the graph of $r = 1 + \cos\theta$
4. Given that $\sinh x = -\frac{3}{4}$. Find $\cosh x$.
5. Show that $\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$ for all real values of x.
6. Find $\lim\limits_{n\to\infty} \sin\left(\dfrac{\pi n}{2n+1}\right)$.
7. Discuss the convergence of \$$\displaystyle\sum_{i=0}^{\infty} \frac{i}{1+i}$
8. Test for convergence $\displaystyle\sum_{i=1}^{\infty} \frac{1}{2^i - i}$
9. Define absolute and conditional convergence of series with example.
10. Find an unit vector in the direction of the vector (3, 1, 2, -1) in R^4.
11. If $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 5$. Evaluate $\det A$ where $A = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ b_1 & b_2 & b_3 \\ c_1 - a_1 & c_2 - a_2 & c_3 - a_3 \end{bmatrix}$
12. Find the values of a and b so that the matrix $\begin{bmatrix} \frac{3}{5} & a \\ \frac{4}{5} & b \end{bmatrix}$ is orthogonal.

(Ceiling: 20 Marks)

Part B (Short essay questions - Paragraph)

Answer ***all*** questions. Each question carries 5 marks.

13. Find the length of the graph of the function $f(x) = \dfrac{x^4}{8} + \dfrac{1}{4x^2}, \quad 1 \le x \le 3.$
14. Show that $\displaystyle\int_0^{\infty} \frac{\sin x}{(1+x)^2}\, dx$ converges.

15. Suppose you are given the following table of data

x	0	0.2	0.4	0.6	0.8	1.0	1.2	1.3	1.4	1.5	1.6	1.7	1.8
$f(x)$	2.037	1.980	1.843	1.372	1.196	0.977	0.685	0.819	1.026	0.799	0.662	0.538	0.555

Evaluate $\int_0^{1.8} f(x)\, dx$ using trapezoidal rule.

16. The set $B = \{u_1, u_2\}$, where $u_1 = \langle 3, 1\rangle,\ u_2 = \langle 1, 1\rangle$ is a basis for R^2. Transform B into an orthonormal basis.

17. Determine whether the set of vectors $u_1 = \langle 2, 6, 3\rangle,\ u_2 = \langle 1, -1, 4\rangle,\ u_3 = \langle 3, 2, 1\rangle,\ u_4 = \langle 2, 5, 4\rangle$ is linearly dependent or linearly independent.

18. Find the eigen values and the correponding eigen vectors of the matrix $A = \begin{pmatrix} 4 & 2 \\ 7 & -1 \end{pmatrix}$. Then without finding A^{-1} find its eigen values and corresponding eigen vectors.

19. State Cayley-Hamilton theorem. Using Cayley-Hamilton theorem find A^{-1} if $A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$

(Ceiling: 30 Marks)

Part C (Essay questions)

Answer any ***one*** question. The question carries 10 marks.

20. Solve the linear system either using Gaussian elimination or Gauss-Jordan elimination $x_1 + 2x_2 + 2x_3 = 2$ $x_1 + x_2 + x_3 = 0$ $x_1 - 3x_2 - x_3 = 0$

21. Find an orthogonal matrix P that diagonalizes the symmetric matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Also find the diagonal matrix D such that $D = P^T AP$.

(1 × 10 = 10 Marks)

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