25U206 (Pages: 2) Name : .....

Reg. No : .....

SECOND SEMESTER UG DEGREE EXAMINATION, APRIL 2026

(FYUGP)

(Regular/Supplementary/Improvement)

CC24UMAT2MN104 - GRAPH THEORY AND AUTOMATA

(Mathematics - Minor Course)

(2024 Admission onwards)

Time: 2.0 Hours Maximum: 70 Marks

Credit: 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 3 marks.

1. Define a Graph. Draw a graph with 5 vertices and 7 edges. [Level:1] [CO1]

2. Define degree of a vertex. Draw a graph G with a vertex of degree 2 and a vertex of degree 3. [Level:2] [CO1]

3. What is a wheel graph. Draw a wheel graph with 5 vertices. [Level:1] [CO1]

4. Show that there is a path between any two distinct vertices in a connected graph. [Level:1] [CO2]

5. Define an Eulerian graph with an example. [Level:2] [CO2]

6. State Kuratowski's theorem. [Level:1] [CO4]

7. Define a planar graph. Provide the Euler's formula of planar graphs. Validate the formula with an example. [Level:2] [CO4]

8. Find the language generated by the grammar $G = (N, T, P, \sigma,)$ where [Level:2] [CO5]
$N = \{\sigma, A, B\}, T = \{a, b\}, P = \{\sigma \to aAa, A \to bBb, \sigma \to \lambda, A \to a, B \to a, B \to b\}$

9. What are the features of phrase structure grammar? [Level:1] [CO5]

10. Let A be a language over $\sum$. Then identify $A\phi$. [Level:2] [CO5]

(Ceiling: 24 Marks)

Part B (Paragraph questions/Problem)

Answer ***all*** questions. Each question carries 6 marks.

11. (a) Given the graph G with vertices $\{1, 2, 3, 4\}$ and edges $\{(1, 2), (2, 3), (3, 4), (4, 1)\}$ determine if the graph is bipartite. Justify your answer by showing how you would divide the vertices into two sets. [Level:2] [CO1]

(b) Explain a complete bipartite graph. Draw a $K_{5,4}$.

12. (a) What does it mean for two graphs to be isomorphic? Provide an example of two isomorphic graphs with 5 vertices. [Level:2] [CO1]

(b) Prove or disprove: "If two graphs have the same number of vertices and edges, they are necessarily isomorphic." Justify your answer with an example.

13. (a) Given a subgraph $G = (V, E)$, explain what is meant by a subgraph of G. Provide an example using a graph with 4 vertices. [Level:2] [CO1]

(b) Can a complete graph be a subgraph of another graph? Justify your answer with an example.

14. Determine if each complete bipartite graph is Hamiltonion. (a) $K_{2,3}$ (b) $K_{3,3}$ (c) $K_{2,4}$ [Level:2] [CO2]

15. Define the following terms with an example: [Level:2] [CO2]

1. Walk 2. Path 3. Closed path 4. Open path 5. Cycle 6. Circuit

16. (a) What is chromatic number of a graph? What is the chromatic number of a cycle graphs C_5 and C_6. [Level:2] [CO4]

(b) Find the chromatic number of the complete graph. Justify your answer.

17. (a) Define a tree, a pendant vertex and draw a tree with 8 vertices among which at least 3 are pendant vertices. [Level:2] [CO4]

(b) Determine the number of edges in a a tree with n vertices.

18. Compute the length of each word over {a, b} 1. aab 2. $aabbb$ 3. ab^4 4. a^3b^2 [Level:2] [CO5]

(Ceiling: 36 Marks)

Part C (Essay questions)

Answer any ***one*** question. The question carries 10 marks.

19. (a) Given a graph with vertices A, B, C, D, and E with the following edges and weights: $AB - (1), AC - (4), BC - (2), BD - (7), CE - (3), DE - (5)$. Apply Kruskal's algorithm to identify the edges in the MST (Minimum spanning tree) and calculate the total weight. [Level:2] [CO4]

(b) Consider a graph with vertices M, N, O, P, and Q and the following edges: $MN - (1),\ MO - (3),\ NO - (4),\ NP - (2),\ OP - (5),\ PQ - (6)$ Apply Kruskal's algorithm to determine the edges in the Minimum Spanning tree (MST) and calculate the total weight.

20. (a) Define a FSA [Level:2] [CO5]

(b) Draw the transition diagram of the FSA $M = (S, A, I, f, s_0)$, where $S = \{s_0, s_1, s_2\}, A = \{s_2\}, I = \{a, b\}$, and the transition function f is defined by $f(s_0, a) = s_1, f(s_0, b) = s_0, f(s_1, a) = s_2, f(s_1, b) = s_0, f(s_2, a) = s_2, f(s_2, b) = s_0$.

(1 × 10 = 10 Marks)

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