23U602 (Pages: 2) Name: ....

Reg.No: ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2026

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19UMTS6B10 / CC20UMTS6B10 - REAL ANALYSIS

(Mathematics - Core Course)

(2019 Admission onwards)

Time : 2.5 Hours Maximum : 80 Marks

Credit : 5

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Give an example to show that a continuous function does not necessarily have an absolute maximum or an absolute minimum on a set.
2. Let f be continuous on the interval $[0, 1]$ such that $f(0) = f(1)$. Prove that there exists a point c in I such that $f(c) = f(c + \frac{1}{2})$.
3. Prove that every Lipschitz function is uniformly continuous.
4. State Weierstrass approximation theorem.
5. Define norm of the partition.Calculate the norm of the partition $P = (1, 3, 6, 7.8, 10)$ of $[1, 10]$.
6. If $f \in \mathfrak{R}[a, b]$, then prove that the value of the integral is uniquely determined.
7. State fundamental theorems of calculus (first and second forms).
8. Define null set. Give an example.
9. Differentiate pointwise convergence and uniform convergence.
10. Define uniform norm. Find the uniform norm of the function f on $[1, 3]$ where $f(x) = x^2$.
11. State Weierstrass M-test.
12. Examine the convergence of the integral $\int_2^\infty \frac{dx}{\ln x}$.
13. Examine the convergence of $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$.
14. Prove that Beta function is symmetric.
15. Express $\int_0^1 x^m(1 - x^2)^n dx$ in terms of Beta function, where $m > 1$ and $n > -1$.

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Prove that Thomae function is continuous at every irrational number and is discontinuous at every rational number in the set of all positive real numbers.

17. State and prove Bolzano's Intermediate value theorem.

18. Let f and g are in $\mathfrak{R}[a,b]$. Prove that $f+g \in \mathfrak{R}[a,b]$ and $\int_a^b (f+g) = \int_a^b f + \int_a^b g$.

19. Prove that if $f : [a,b] \to \mathbb{R}$ is a monotone function, then $f \in \mathfrak{R}[a,b]$.

20. Prove Cauchy criterion for uniform convergence of sequence of functions.

21. Let (f_n) be a sequence of continuous functions on $A \subseteq \mathbb{R}$ and suppose that (f_n) converges uniformly to a function $f : A \to \mathbb{R}$. Then prove that f is continuous on A.

22. Find the Cauchy principal value of $\int_{-1}^{5} \frac{dx}{(x-1)^3}$.

23. Prove that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. Let f be a continuous function on a closed and bounded interval I. Prove that (i) f is bounded on I (ii) f has an absolute maximum on I.

25. Let $f : [a,b] \to \mathbb{R}$ and $c \in (a,b)$. Then prove that $f \in \mathfrak{R}[a,b]$ if and only if its restrictions to $[a,c]$ and $[c,b]$ are both Riemann integrable and $\int_a^b f = \int_a^c f + \int_c^b f$.

26. Check whether $\int_{\pi}^{\infty} \frac{\sin x}{x}\, dx$ is conditionally convergent.

27. Establish the relation between Beta and Gamma function.

(2 × 10 = 20 Marks)

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