23U603 (Pages: 2) Name: ....

Reg.No: ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2026

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19UMTS6B11 / CC20UMTS6B11 - COMPLEX ANALYSIS

(Mathematics - Core Course)

(2019 Admission onwards)

Time : 2.5 Hours Maximum : 80 Marks

Credit : 5

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Using properties of limit evaluate $\lim_{z \to 1+i} (z^2 + i)$
2. Show that the function $f(z) = \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}$ is analytic in any domain that does not contain origin.
3. Show that $f(z) = y + ix$ is nowhere analytic.
4. Prove that $e^{z_1+z_2} = e^{z_1}e^{z_2}$
5. Find all possible values of $\ln(1+i)$.
6. Express the value of $\sin 4i$ in the form $a + ib$.
7. Find $\frac{d}{dz}(\sin z \sinh z)$.
8. Evaluate $\int_C ydx + xdy$ where C is the graph of $y = x^2$ from $(0,0)$ to $(1,1)$.
9. Evaluate $\oint_C \frac{2z+1}{z^2+z}dz$ where C is the circle $|z - 3i| = 1$.
10. Evaluate $\int_{\pi}^{\pi+2i} \sin(z/2)dz$
11. State Cauchy's integral formula.
12. Determine whether the sequence $\left\{\frac{3+ni}{n+2ni}\right\}$ converges or diverges.
13. Use the sequence of partial sums to show that the series $\sum_{k=1}^{\infty}\left[\frac{1}{k+2i} - \frac{1}{k+1+2i}\right]$ converges.
14. Define the term 'isolated singularity'. Give a classification with examples.

15. Let $f(z) = \dfrac{2}{(z-1)(z+4)}$, find $\text{Res}(f(z), 1)$.

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Using definition find $f'(z)$, given $f(z) = z - \dfrac{1}{z}$

17. Show that the function $u(x, y) = xy + x + 2y$ is harmonic in the entire complex plane. Also find the harmonic conjugate function *v* and find analytic function $f(z) = u + iv$ satisfying the condition $f(2i) = -1 + 5i$

18. State the bounding theorem. Use it to find an upper bound for the absolute value of the integral $\displaystyle\int_C \frac{e^z}{z+1} dz$ where C is the circle $|z| = 4$.

19. Evaluate $\displaystyle\oint_C \frac{\cos 2z}{z^5} dz$ where C is the circle $|z| = 1$.

20. State and prove Cauchy's inequality.

21. Find the circle and radius of convergence of the power series $\displaystyle\sum_{k=0}^{\infty} (1+3i)^k (z-i)^k$

22. Expand $f(z) = \dfrac{1}{z(z-1)}$ in a Laurent series valid for $1 < |z-2| < 2$

23. State residue theorem. Using residue theorem evaluate $\displaystyle\oint_C \frac{2z+6}{z^2+4} dz$ where C is the circle $|z - i| = 2$.

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. By assuming $f(z) = u + iv$ is differentiable, derive Cauchy-Riemann equations.

25. Evaluate $\displaystyle\int_C \text{Im}(z-i) dz$ where C is the polygonal path consisting of the circular arc along $|z| = 1$ from $z = 1$ to $z = i$ and the line segment from $z = i$ to $z = -1$.

26. State and prove Taylor's theorem.

27. Evaluate $\displaystyle\int_0^{2\pi} \frac{1}{(2+\cos\theta)^2} d\theta$.

(2 × 10 = 20 Marks)

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