23U604 (Pages: 2) Name: ....

Reg.No: ....

SIXTH SEMESTER B.Sc DEGREE EXAMINATION, APRIL 2026

(CBCSS - UG)

(Regular/Supplementary/Improvement)

CC19UMTS6B12 / CC20UMTS6B12 - CALCULUS OF MULTIVARIABLE

(Mathematics - Core Course)

(2019 Admission onwards)

Time : 2.5 Hours Maximum : 80 Marks

Credit : 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Let $f(x,y) = x^2 - xy + 2y$. Find the domain of f and evaluate $f(x+y, x-y)$.
2. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ if $f(x,y) = 2x^2y^3 - 3xy^2 + 2x^2 + 3y^2 + 1$.
3. Find h_w if $h(x,y,z,w) = \dfrac{xw^2}{y + \sin(zw)}$.
4. Find $\frac{dy}{dx}$ if $x^4 + 2x^2y^2 - 3xy - x = 5$.
5. Let $f(x,y) = x^2 + y^2 - 2x + 4y$. Find the critical point of f.
6. Evaluate$\int_0^2 \int_1^4 y\sqrt{x}\, dydx$.
7. Evaluate $\int\int_R (x+2y)dydx$, where $R = \{(x,y) : 0 \le x \le 1, 0 \le y \le x\}$.
8. Evaluate$\int_1^4 \int_0^1 \int_0^2 x^2 dxdydz$
9. Evaluate $\int_0^\pi \int_0^2 \int_0^1 rdzdrd\theta$
10. Find the jacobian of the transformations T defined by the equations $x = u^2 - v^2, y = 2uv$.
11. Find the gradient vector field of $f(x,y,z) = x^2 + xy + y^2z^3$.
12. Find the $curl F(-1,2,1)$, if $F(x,y,z) = xy\hat{i} + xz\hat{j} + xyz^2\hat{k}$.
13. State Green's theorem.
14. Find a parametric representation of the plane passing through the point $(3,-1,1)$ and containing the vectors $a = -2\hat{i} + 5\hat{j} + \hat{k}$ and $b = -3\hat{i} + 2\hat{j} + 3\hat{k}$.
15. State divergence theorem.

(Ceiling: 25 Marks)

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. A storage tank has the shape of a right circular cylinder. Suppose that the radius and height of the tank are measured at 1.5 ft and 5 ft, respectively, with a possible error of 0.5 ft and 0.1 ft respectively. Use differentials to estimate the maximum error in calculating the capacity of the tank.

17. Find the directional derivative of $f(x,y,z) = x^2e^{yz}$ at the point $(2,3,0)$ in the direction of $v = i - 2j + 3k$.

18. Find equations of the tangent plane and normal line to the surface with equation $2x^2 - y^2 + 3z^2 = 2$at the point $(2,-3,1)$.

19. Find the volume of the solid S that lies below the hemisphere $z = \sqrt{9 - x^2 - y^2}$, above the xy-plane and inside the cylinder $x^2 + y^2 = 1$.

20. Find the mass of the lamina occupying a triangular region R with vertices $(0,0)$, $(2,0)$ and $(0,2)$ if its mass density at a point (x,y) in R is $\rho(x,y) = x + 2y$.

21. Find the surface area of the part of the paraboloid $z = 9 - x^2 - y^2$ that lies above the plane $z = 5$.

22. Evaluate $\int_C ydx + xdy$, where C is the line segment from $(1,-1)$ to $(4,2)$.

23. Find the mass of the surface S composed of the part of the paraboloid $y = x^2 + z^2$ between the planes $y = 1$ and $y = 4$ if the density at a point P on S is inversely proportional to the distance between P and the axis of symmetry of S.

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. (i) Show that $\lim_{(x,y)\to(0,0)} \dfrac{2xy}{\sqrt{x^4 + y^4}}$ does not exist.

    (ii) Using $\varepsilon - \delta$ definition of limits, prove that $\lim_{(x,y)\to(0,0)} \dfrac{2x^2y}{x^2 + y^2} = 0$.

25. Find the absolute extreme values of $f(x,y) = 2x^2 + y^2 - 2y + 1$ subject to the constraint $x^2 + y^2 \leq 4$.

26. Let $F(x,y,z) = 2xyz^2\hat{i} + x^2z^2\hat{j} + 2x^2yz\hat{k}$.

    (a) Show that F is conservative, and find a potential function f such that $F = \nabla f$.

    (b) If F is a force field, find the work done by F in moving a particle along any path from $(0,1,0)$ to $(1,2,-1)$.

27. Verify Stockes' Theorem for the vector field $F(x,y,z) = y\hat{i} + z\hat{j} + x\hat{k}$, where S is the part of the plane $2x + 2y + z = 6$ lying in the first octant, oriented with the normal pointing outward.

(2 × 10 = 20 Marks)

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