15U112

Name:............................
Reg. No.........................

FIRST SEMESTER DEGREE EXTERNAL EXAMINATION DEC./JAN. 2015-16'

(2015 Admission)

CC15UMAT1B01– FOUNDATIONS OF MATHEMATICS(core)

Time: Three Hours

Maximum: 80 Marks

I. **Answer *all* questions:**

1. Find the domain of f(x) = $\sqrt{1+x}$
2. What is the power set of $\{\varphi\}$
3. If $\lfloor x \rfloor$ denote the integer floor function at x then $\lfloor -12.25 \rfloor$ =......
4. Find -10(mod 3)
5. If $\sqrt{5-2x^2} \le f(x) \le \sqrt{5-x^2}$ for -1 ≤ x ≤1, find $\lim_{x\to 0} f(x)$.
6. Cardinal number of the infinite set P of positive integers is ....
7. The dual of the preposition (p ∨ F) ∧ (q ∨ T) is ....
8. $\lim_{x\to 0} x\sin\frac{1}{x}$ = ...
9. Give an example of nested sequence of open intervals whose intersection is empty.
10. The Cartesian product of A = {1, 2} and B = { a, b, c} is ....
11. Define an algebraic number.
12. The points of discontinuity of the function f(x) = $\frac{x+2}{\cos x}$ are ....

(12 × 1 = 12 Marks)

II. **Answer any nine questions.**

13. Let f: R→R and g: R→R be defined by f(x) =x^2 and g(x) =x+3 .Compute f∘g and g∘f.
14. Show that the preposition p → q and ¬p ∨ q are logically equivalent.
15. Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c, d\}$ and let $R = \{(1, a), (2, d), (3, a), (3, b), (3, d)\}$. Find the matrix of the relation.
16. Given that f: A → B and g: B → C are injective, Prove that the composite function g∘f is injective.
17. Find the centre and radius of the circle circle $x^2 + y^2 - 4x - 4y + 4 = 0$
18. For what value of a, is $f(x) = \begin{cases} x^2-1 & x<3 \\ 2ax & x\ge 3 \end{cases}$ continuous at every x.
19. Prove that f(x) = |x| is continuous at every value of x

24. Let A = {1,2,3........14,15} and R be a ternary relation on A defined by the equation $x^2 + 5y = z$. Write R as a set of ordered triples.

(9 × 2 = 18 mark

III. Answer any six questions.

25. Prove that $\lim_{x \to 2} f(x) = 4$ if $f(x) = \begin{cases} x^2, & x \neq 2 \\ 1, & x = 2 \end{cases}$

26. Express the statement "Some students in this class has visited Mumbai" and "Every students in this class has visited either Chennai or Mumbai" using quatifiers.

27. Show that the product set P×P is countably infinite.(P is the set of positive integers).

28. Show that the prepositions $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

29. Check whether x/y (x divides y) on the set P of positive integers is

(a) Reflexive (b)Symmetric (c)Transitive

30. Define h(2) in a way that extends $h(t) = \frac{t^2+3t-10}{t-2}$ to be continuous at t=2.
31. Find all the partitions of S ={a,b,c,d}.
32. Graph the parabola $y = x^2 - 2x - 3$. Label the vertex , axis and intercepts, if any.
33. Graph the function let $f(x) = \begin{cases} x, & x < 1 \\ x+1, & x > 1 \end{cases}$

Discuss the behaviour of the function as $x \to 1$.

(6 × 5 = 30 Mark

IV. Answer any two questions.

34. Express the following statements into logical expression using quantifiers?
    (a) "All humming birds are richly coloured"
    (b) "No large birds live on honey"
    (c) "Birds that do not live on honey are dull in colour"
    (d) "humming birds are small"
35. (a)Define an equivalence relation on a non empty set ,equivalence class , and the quotient set.

    (b) A= {1,2,3,.......14,15} ,the relation on $A \times A$ defined by $(a,b) \sim (c,d)$ iff $ad = bc$ prove that '~' is an equivalence relation.Find th equivalence class of (3,2).
36. (a)Do $\lim_{x \to 0} f(x)$ exist, if $f(x) = \frac{x}{|x|}$

    (b)Evaluate $\lim_{\theta \to 3^+} \frac{\lfloor \theta \rfloor}{\theta}$ and $\lim_{\theta \to 3^-} \frac{\lfloor \theta \rfloor}{\theta}$ where $\lfloor \theta \rfloor$ is the integer floor function

    (c) Evaluate $\lim_{x \to 0^+} \frac{x^2}{2} - \frac{1}{x}$

(2 × 10 = 20 Mar

****