FDTD Analysis of the Interaction of a Gaussian Pulse with Negative Permittivity Metamaterial Slab

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***Abstract*—The interaction of a Gaussian pulse with a negative permittivity metamaterial slab is presented. The results of this numerical study predict the propagation behavior of electromagnetic wave in presence of the slab. To perform this analysis the Auxiliary Differential Equation Finite Difference Time Domain (ADE-FDTD) analysis for a dispersive medium using Drude model is used. The negative permittivity plasma medium is modeled using effective medium theory. A Gaussian pulse of wide frequency bandwidth is used for the analysis. The field distributions for frequencies above and below the plasma frequency are investigated. The study reveals the frequency selective properties of the negative permittivity slab in relation to the reflection and transmission characteristics.**

***Index Terms*—Dispersive medium, Drude model, FDTD, Gaussian pulse, Metamaterials, Negative Permittivity.**

I. Introduction

Metamaterials are engineered periodic structures having homogeneous properties for the interacting waves. These materials show unusual electromagnetic properties not observed with normal materials. The unusual behaviors of these materials are due to its negative values of permeability μ, permittivity ϵ and index of refraction n. Periodic structures having negative μ and ϵ are mixed together to get negative n structure [1]–[4]. Finite Difference Time Domain (FDTD) method offers a simple and straight forward way for modeling complicated periodic structures [5]–[9] like metamaterial and photonic band-gap structures. Since these artificial plasma media consists of periodic distribution of conducting elements, it is dispersive [10]–[13]. There are two possibilities for the modeling of this medium in FDTD. One is the actual insertion of conducting elements periodically in the computational domain by assigning high electron density for the selected cells and the other is by considering the medium in terms of effective medium properties. Mainly three methods are used for the analysis of a dispersive medium using FDTD [5]. They are Auxiliary differential Equation method (ADE), Recursive Convolution method (RC), and the Z-transform method.

The electromagnetic wave propagation in presence of a negative permittivity slab using effective medium theory is preformed in this study. A Gaussian pulse of wide frequency bandwidth is used to illuminate the computational domain. The signal strengths in front and back of the negative permittivity slab for different frequencies are analyzed.

II. Formulation of the Problem

The problem is simulated in 2-dimensional dispersive FDTD space using effective medium theory. The model equations involved in 2D-ADE-FDTD are used. In this study Drude model is used for modeling frequency dependent permittivity slab. According to this model, the relative permittivity ϵ_r (ω) and relative permeability μ_r (ω) are as follows,

$$\epsilon_r(\omega) = 1 - \frac{\omega_{ep}^2}{\omega^2 - \omega_{e0}^2 - i\gamma_e\omega} \tag{1}$$

$$\mu_r(\omega) = 1 - \frac{\omega_{mp}^2}{\omega^2 - \omega_{m0}^2 - i\gamma_m\omega} \tag{2}$$

where ω_{ep} is the electric plasma frequency and ω_{e0} is the low frequency edge of the electric forbidden band. γ_e is the electric damping factor. ω_{mp}, ω_{m0} and γ_m are the corresponding factors for the magnetic field. For a TE wave with field components E_z, H_x and H_y, Maxwells curl equations in component form are given by,

$$\frac{\partial D_z}{\partial t} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \tag{3}$$

$$\frac{\partial B_x}{\partial t} = -\frac{\partial E_z}{\partial y} \tag{4}$$

and

$$\frac{\partial B_y}{\partial t} = \frac{\partial E_z}{\partial x} \tag{5}$$

The expressions for D and B in terms of the components of electric and magnetic fields E_z, H_x and H_y can be found out using Maxwells equations and Drude model as [7],

$$D_z = \epsilon_0(1 - \frac{\omega_{ep}^2}{\omega^2 - \omega_{e0}^2 - i\gamma_e\omega})E_z \tag{6}$$

$$B_x = \mu_0(1 - \frac{\omega_{mp}^2}{\omega^2 - \omega_{m0}^2 - i\gamma_m\omega})H_x \tag{7}$$

$$B_y = \mu_0(1 - \frac{\omega_{mp}^2}{\omega^2 - \omega_{m0}^2 - i\gamma_m\omega})H_y \tag{8}$$

Equations used for modeling of metamaterials are obtained by simplifying and converting to time domain by replacing $i\omega$ by $\partial/\partial t$ and applying the second order FDTD discretization to the Eqn. 6 - Eqn. 8.

The computational domain is divided into three regions and a negative permittivity slab of thickness d is defined in it as

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shown in Fig. 1. Source point is defined in front of the slab in region III as shown. In region I ($0 < x < x^1$) and III ($x^1 + d < x < x_{max}$), that is the regions behind and in front of the slab, the electric plasma frequency ω_{ep} and magnetic plasma frequency ω_{mp} are taken as zero. Hence permittivity and permeability of these regions are always positive. In region II ($x^1 < x < x^1 + d$), where the slab is defined, the electric plasma frequency ω_{ep} is taken as $2\pi(10 \times 10^9)$ and magnetic plasma frequency ω_{mp} is taken as zero. Hence according to Drude model, permittivity of the slab is negative below 10 GHz and permeability is always positive. That means, the permittivity of the slab depends on the frequency of the interacting electromagnetic wave. Here damping factors are not considered. Hence γ_e and γ_m are taken as zero. The 2D-FDTD space used is of dimension 650×650 cells and the negative permittivity slab defined is between cells 210 to 280 along X and 180 to 480 along Y. Grid size for X and Y directions are 1.5 mm each. The time step is set to be $\Delta t = \Delta x / \sqrt{2}c$. A time step of 500 is observed to be sufficient to evaluate the frequency distribution in front and behind the slab. Gaussian pulse is excited at 30^{th} grid point in front of the slab. Measurement is taken from 55 and 205 cells behind and in front of the center of the slab.

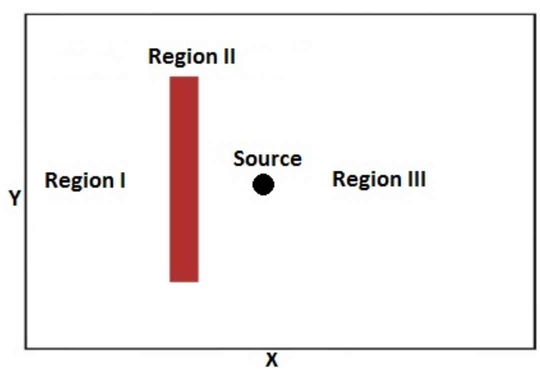

Fig. 1. Schematic representation of the computational domain with three regions and a Gaussian source defined in front of the slab.

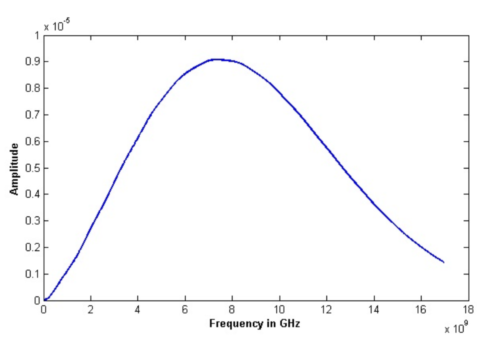

Fig. 2. Frequency spectrum of the Gaussian pulse used to excite the computational domain.

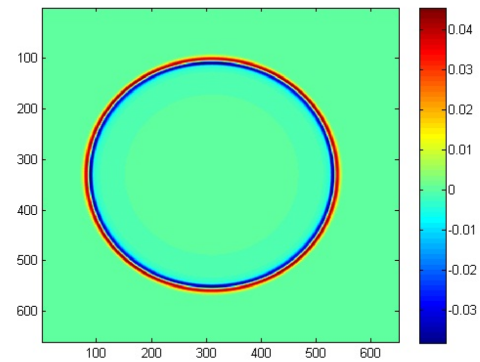

Fig. 3. Propagation of the Gaussian pulse through the work space in the absence of the negative permittivity slab. The excitation point is x = 310, y = 330.

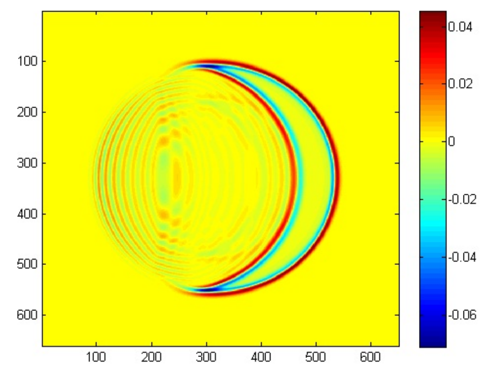

Fig. 4. Propagation of the Gaussian pulse through the work space in presence of the negative permittivity slab. The excitation point is x = 310, y = 330. The position of the slab is between x = 210-280 and y = 180-480.

III. Results and Discussions

A Gaussian pulse with its amplitude maximum of the frequency distribution around the selected plasma frequency is used to excite the work space. In this case the electrical plasma frequency selected is f = 10 GHz. The frequency spectrum of the Gaussian pulse used is plotted in Fig. 2 The propagation of the Gaussian pulse through the computational domain with and without the negative permittivity slab are shown in Fig. 3 and Fig. 4. In Fig. 5 normalized intensity in dB of the power transmitted for different frequencies through the slab with plasma frequency 10 GHz is presented. It is evident from the figure that the intensity in the rear side of the slab is less, which is due to the reflection or attenuation of frequencies below the plasma frequency. Complete transmission of power through the slab is observed around 12 GHz. Fig. 6 is the intensity distribution for the front side of the slab and all frequencies are found to be present here. Some frequencies below 12 GHz are constructively interfered along the axis of the source and some others are interfered destructively, which is observed as the variations in the intensity.

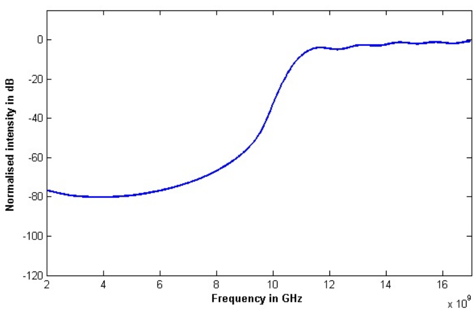

Fig. 5. Normalised intensity behind the slab corresponding to x = 190, y = 330.

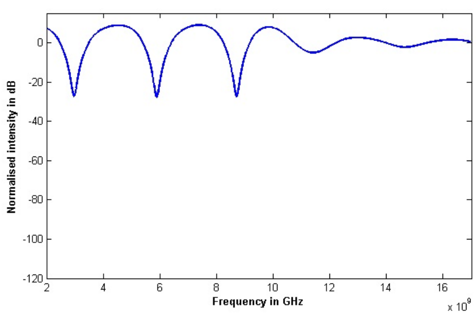

Fig. 6. Normalised intensity in front of the slab corresponding to x = 450, y = 330.

IV. Conclusion

The propagation of a Gaussian pulse through a dispersive medium in presence of a negative permittivity slab is analyzed using dispersive 2D-ADE FDTD method by considering it as an effective medium. The propagation of electromagnetic waves of different frequencies are analyzed by considering the Fourier Transform of the power spectrum. The study shows that frequencies below the plasma frequency of the negative permittivity slab are reflected or attenuated and frequencies above that are transmitted. The results predict the possibility of changing the field distribution around the plasma slab by varying the frequency of the excitation source. It may also be helpful for the design and fabrication of novel artificial plasma reflector antenna systems having frequency dependent radiation properties.

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