16U406 (Pages:3) Name: ……………..………

Reg. No……………..…………..

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2018

(Regular/Supplementary/Improvement)

(CUCBCSS - UG)

CC15U MAT4 B04 – THEORY OF EQUATIONS, MATRICES AND VECTOR CALCULUS

(Mathematics - Core Course)

(2015 Admission onwards)

Time: Three Hours Maximum: 80 Marks

PART - A

Answer **All** Questions. Each question carries **one** mark.

1. If α, β and γ are the roots of the equation $x^3 - x - 1 = 0$, find the equation whose roots are $\alpha + 1$, $\beta + 1$ and $\gamma + 1$.
2. Form an equation whose roots are the negatives of the roots of the equation $2x^3 - 5x^2 + 7 = 0$.
3. Give an example of a standard reciprocal equation.
4. State Descarte's rule of signs.
5. Find the rank of the matrix $A = \begin{bmatrix} 2 & 4 & 3 & 2 \\ 3 & 3 & 1 & 4 \end{bmatrix}$.
6. If $A = [a_{ij}]$ is an $m \times n$ matrix and $a_{ij} = 2$, for all i, j. Find the rank of A.
7. Compute the product $E_{21}(p)\, E_{31}(q)$ of the elementary matrices of order 3.
8. Find the value of 'p' if the system of equations $2x + y = 5$, $4x + 2y = p$ has infinitely many solution if p is:

   (a) 1 (b) 5 (c) 10
9. If λ is a characteristic root of a non-singular matrix A, then characteristic root of A^{-1} is -------
10. Find the parametric equation of a line through the point $(3, -4, -1)$ and parallel to the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$.
11. The name of the surface whose equation is $\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$
12. Find the cylindrical coordinates of the cylinder whose Cartesian equation is $x^2 + y^2 = 2x$.

(12 x 1 = 12 Marks)

 Turn Over

PART - B

Answer any ***nine*** Questions. Each question carries 2 marks.

13. Solve the equation $4x^4 - 8x^3 + 7x^2 + 2x - 2 = 0$ of which one root is $1 + i$.
14. If α, β and γ are the roots of the equation $x^3 + ax^2 + bx + c = 0$, find the equation whose roots are $\alpha\beta, \beta\gamma$ and $\alpha\gamma$.
15. If α, β and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \frac{1}{\alpha}$.
16. Find the least number of imaginary roots of the equation $x^9 + 5x^8 - x^3 + 7x + 2 = 0$.
17. Find the rank of $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$ by reducing it to its normal form.
18. Test whether the system of equations $\begin{matrix} 2x - 4y = 3 \\ -3x + 6y = -4 \end{matrix}$ are consistent.
19. Find the value of k, if the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 1 & k \end{bmatrix}$ is 2.
20. Find the characteristics roots of $\begin{bmatrix} 3 & -4 \\ 2 & -6 \end{bmatrix}$.
21. Show that the eigen values of a diagonal matrix are the same as its diagonal elements.
22. If $3\hat{\imath} + 4\hat{\jmath}$, $2\hat{\imath} + 3\hat{\jmath} + +4\hat{k}$ and $5\hat{k}$ are the vectors representing sides of a parallelepiped at one corner, find its volume.
23. Find the distance of the point $(2,3,0)$ from the plane $3x + 2y - 6z + 9 = 0$.
24. Evaluate $\int_{-\pi/4}^{\pi/4} \left[(\sin t)\hat{\imath} + (1 + \cos t)\hat{\jmath} + (\sec^2 t)\,\hat{k}\right] dt$.

(9 x 2 = 18 Marks)

PART - C

Answer any ***six*** Questions. Each question carries 5 marks.

25. Frame an equation with rational coefficients, one of whose roots is $\sqrt{5} + \sqrt{2}$.
26. If α, β and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $(\alpha^2 + \beta\gamma) + (\beta^2 + \alpha\gamma) + (\gamma^2 + \alpha\beta)$.
27. Solve the equation $x^5 - 6x^4 + 7x^3 + 7x^2 - 6x + 1 = 0$.
28. For the matrix A, find non-singular matrices P and Q such that PAQ is in normal form, where $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$.

29. Using matrix method solve the system of equations $\begin{matrix} x + 2y + z = 3 \\ 2x + 5y - z = -4 \\ 3x - 2y - z = 5 \end{matrix}$.
30. Show that 4 is an eigen value of $\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$ and find a corresponding eigen vector.
31. State Cayely-Hamilton theorem and verify the Cayely-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.
32. Solve the initial value problem: $\frac{dr}{dt} = -t\mathbf{i} - t\mathbf{j} - t\mathbf{k}$, $\mathbf{r}(0) = \mathbf{i} + 2\,\mathbf{j} + 3\mathbf{k}$.
33. Find T, N and κ for the space curve $\mathbf{r}(t) = (a\cos t)\,\mathbf{i} + (a\sin t)\,\mathbf{j} + bt\mathbf{k}$, where $a, b \geq 0$, $a^2 + b^2 \neq 0$.

(6 x 5 = 30 Marks)

PART - D

Answer any ***two*** Questions. Each question carries **ten** marks.

34. (a) Solve the equation $81x^3 - 18x^2 - 36x + 8 = 0$, whose roots are in harmonic progression.

    (b) Solve the cubic equation $x^3 - 9x + 28 = 0$ by Cardano's method.
35. Using elementary transformations find the inverse of $A = \begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$.
36. (a) Find the point of intersection of the line $\frac{x-1}{3} = \frac{y-2}{1} = \frac{z+1}{2}$ and the plane $4x - y - 5z - 4 = 0$.

    (b) Translate the equation $x^2 + y^2 + z^2 = 4z$ into cylindrical and spherical equations.

(2 x 10 = 20 Marks)

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