C 21072 (Pages : 4) Name..................................

Reg. No..................................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2017

(CUCBCSS—UG)

Mathematics

MAT 6B 11—NUMERICAL METHODS

Time : Three Hours Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*

Each question carries 1 *mark.*

1. Set up a Newton's iteration for computing $\sqrt{2}$.
2. Construct the difference table of $f(x) = x^3 - 3x^2 + 5x + 7$ for the values $x = 0, 2, 4, 6, 8$.
3. Give the Gauss's forward interpolation formula.
4. What do you mean by central differences ?
5. Evaluate $\Delta^n \left(e^x\right)$, interval of differencing being unity.
6. Write the relation between divided differences and forward differences.
7. Given a set of n-values of (x, y), what is the formula for computing $\left[\frac{d^2y}{dx^2}\right]_{x_0}$.
8. State general formula for numerical integration.
9. State Adams-Bashforth formula.
10. What is the order of the error in Simpson's 1/3 - rule.
11. In solving $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$, write down Taylor's series for $y(x_1)$.
12. Write Runge-Kutta formula of fourth order to solve $\frac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$.

(12 × 1 = 12 marks)

Turn over

Section B

Answer any **ten** *out of fourteen questions.*
Each question carries 4 marks.

13. Find a real root of the equation $x^3 - 2x - 5 = 0$ using secant method.

14. Prove that (i) $\Delta = E - I$; (ii) $E = e^{hD}$ where E is the shift operator and D is the differential operator.

15. Find the missing term in the following table :

x ...	0	1	2	3	4
y ...	1	3	9	-	81

16. Find the divided differences of $f(x) = x^2 + x + 2$ for the arguments 1, 3, 6, 11.

17. Prove that n^{th} divided differences of a polynomial of n^{th} degree are constants.

18. Using Lagrange's interpolation formula, find the form of the function $y(x)$ from the following table :

x ...	0	1	3	4
y ...	−12	0	12	24

19. The speed, v meters per second, of a car, t seconds after it starts, is shown in the following table :

t	0	12	24	36	48	60	72	84	96	108	120
v	0	3.60	10.08	18.90	21.60	18.54	10.26	5.40	4.50	5.40	9.00

Find the distance travelled by the car in 2-minutes.

20. Derive Simpson's (3/8)-rule $\int_{x_0}^{x_3} y\,dx = \frac{3}{8}h\,(y_0 + 3y_1 + 3y_2 + y_3)$.

21. Explain Simpson's 1/3-rule.

22. Decompose the matrix $\begin{bmatrix} 2 & -3 & 10 \\ -1 & 4 & 2 \\ 5 & 2 & 1 \end{bmatrix}$ in the form LU.

23. From the following table, estimate the number of men getting wages between 100 and 150 :

Wages in Rupees	0–100	100–200	200–300	300–400
No. of Men	9	30	35	42

24. Solve the system of equations $28x + 4y - z = 32$; $x + 3y + 10z = 24$; $2x + 17y + 4z = 35$ by Gauss-Seidel iteration method .

25. Using Picard's method obtain a solution upto the fifth approximation to the equation $\frac{dy}{dx} = y + x$, such that $y(0) = 1$.

26. Using Adams-Moulton method , find $y(1.4)$ given

$$\frac{dy}{dx} = x^2 (1 + y), y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548 \text{ and } y(1.3) = 1.979.$$

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of nine questions.*

Each question carries 7 marks.

27. Using Newton's iterative method, find the real root of $x \log_{10} x = 1.2$ correct to five decimal places.

28. Using the method of separation of symbols, show that

$$\Delta^n u_{x-n} = u_x - nu_{x-1} + \frac{n(n-1)}{2} u_{x-2} + \dots + (-1)^n u_{x-n}.$$

29. Find the smallest root of the equation $f(x) = x^3 - 6x^2 + 11x - 6 = 0$.

30. From the following table, find the value of sin $38°$:

x (in degrees)	15	20	25	30	35	40
sin x	0.2588190	0.3420201	0.4226183	0.5	0.5735764	0.6427876

31. Given $\sum_{1}^{10} f(x) = 500426, \sum_{4}^{10} f(x) = 329240, \sum_{7}^{10} f(x) = 175212$ and $f(10) = 40365$ find $f(1)$.

32. Find the first and second derivatives of the function tabulated below at the point $x = 1.5$:

x	1.5	2.0	2.5	3.0	3.5	4.0
$f(x)$	3.375	7.0	13.625	24.0	38.875	59.0

33. Apply Lagrange's formula inversely to obtain a root of the equation $f(x)=0$, given that $f(30)=-30, f(34)=-13, f(38)=3$ and $f(42)=18$.

34. Apply Runge-Kutta method, to find an approximate value of y when $x = 0.2$ given that $\frac{dy}{dx} = x + y, y(0) = 1$.

35. Find the inverse of the matrix $\begin{bmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{bmatrix}$ using Gauss Elimination Method.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 13 *marks.*

36. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using

(a) Trapezoidal rule taking $h=1$.

(b) Simpson's $\frac{1}{3}$ rule taking $h=1$.

(c) Simpson's $\frac{3}{8}$ rule taking $h=1$.

37. Solve the system of equations :

$x_1 + 2x_2 + x_3 - x_4 = -2$; $x_1 + x_2 + 3x_3 - 2x_4 = -6$; $2x_1 + 3x_2 - x_3 + 2x_4 = 7$; $x_1 + x_2 + x_3 + x_4 = 2$

by Gauss Jordan method.

38. (a) Apply Milne's method, to find a solution of the differential equation $\frac{dy}{dx} = x - y^2$ in the range $0 \le x \le 1$ for the boundary condition $y = 0$ at $x = 0$.

(b) Determine the largest eigen value and corresponding eigen vector of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

(2 × 13 = 26 marks)