

16U311

(Pages:3)

Name:

Reg. No.....

THIRD SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2017

(CUCBCSS - UG)

Mathematics - Core Course

CC15U MAT3 B03 - CALCULUS AND ANALYTIC GEOMETRY

(2015 Admission onwards)

Time: Three Hours

Maximum: 80 Marks

Section A

Answer *all* questions. Each question carries 1 mark.

1. Find the simpler expression of the quantity.
2. Find the range of the hyperbolic function
3. Find
4. Give an example of a monotonically decreasing sequence.
5. Find a formula for the n th term of the sequence
6. Find the value of x for which.
7. Express $0.232323\dots$ as the ratio of two integers.
8. Find the radius of convergence of the series.
9. Find the focus of the parabola.
10. Draw the path of the curve $r = 2 + 2\cos\theta$.
11. Replace the polar equation $r = 4\cos\theta$ by equivalent Cartesian equation.
12. What is the polar equation of the circle with centre $(2, \frac{\pi}{3})$ and radius 2?

Section B

Answer Any *Nine* Questions. Each Question Carries 2 Marks.

13. Evaluate $\int_0^1 x^2 dx$.
14. Prove that $\lim_{n \rightarrow \infty} \frac{1}{n} = 0$.
15. Evaluate $\int_0^1 x^2 dx$.
16. Prove that e^x grows faster than x^n .
17. Evaluate $\int_0^1 x^2 dx$.
18. Find the sum of the series $\sum_{n=0}^{\infty} x^n$.
19. Find the Taylor series generated by e^x at $x=0$. Where, if anywhere, does the series converge to?
20. Find the equation of the parabola with focus $(2, 0)$ and directrix $x = -2$.
21. Prove that the equation $x^2 + y^2 = 4$ represents an ellipse.

22. Draw the graph of the curve.
23. Show that the point (a, b) lies on the curve.
24. Find the area of the region in the plane enclosed by the cardioids.

Section C

Answer any *six* questions. Each question carries 5 marks.

25. Find the domain, range, concavity and points of inflection of the function $f(x)$ and draw its graph.
26. Evaluate $\int_0^1 x^2 dx$.
27. Prove that, if $\sum a_n$ converges, then $\sum na_n$ converges.
28. Test for convergence or divergence of the series $\sum \frac{1}{n^2}$.
29. Show that the series $\sum \frac{1}{n^2}$ is conditionally convergent.
30. If $\sum a_n$ converges, find series for $\sum na_n$ and $\sum n^2 a_n$.
31. By a suitable rotation of the coordinate axes remove the cross product term from the equation $4x^2 + 12xy + 9y^2 = 1$ and then identify the graph of the equation.
32. Find the equation of the tangent to the curve $y = x^2 + 2x + 1$ at $(1, 4)$. Also find the value of $\frac{dy}{dx}$ at the same point.
33. Find the points of intersection of the curves $y = x^2$ and $y = x^3$.

Section D

Answer any *two* questions. Each question carries 10 marks.

34. (a) Show that the series $\sum \frac{1}{n^2}$ converges if $x < 1$ and diverges if $x \geq 1$.
 (b) Prove that the series $\sum \frac{1}{n^2}$ converges.
35. Find the length of the asteroid $x^{2/3} + y^{2/3} = 1$. Also find the centroid of the first quadrant arc of the above asteroid.
36. (a) Find the area of surface generated by revolving the right-hand loop of the lemniscates $r^2 = a^2 \cos 2\theta$ about the y axis.
 (b) Find the area shared by the circle $x^2 + y^2 = a^2$ and the cardioids $r = a(1 + \cos \theta)$.
