15U602 (Pages: 2) Name: ……………………

Reg. No……………………

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2018

(CUCBCSS-UG)

CC15U MAT6 B10 - COMPLEX ANALYSIS

Mathematics - Core Course

(2015 Admission)

Time: Three Hours Maximum marks:120

PART – A

Answer ***all*** questions. Each question carries 1 mark.

1. $\lim_{z\to\infty} \frac{2z+i}{z+1}$ = ---
2. Verify Cauchy Riemann equations for the function $f(z) = \cos x \cosh y - i \sin x \sinh y$
3. Prove that $u = \frac{x}{x^2+y^2}$ is harmonic.
4. $\sin(iy)$ = ---
5. State Cauchy Integral formula.
6. $\int_{|z|=2}(z^2 + 5)\, dz$ = ---
7. Every bounded entire function is ---
8. The region of convergence of $1 - z + z^2 - z^3 + \cdots$ is ---------
9. The radius of convergence of $\sum a_n z^n$ is ---
10. Define a simply connected domain.
11. The function $f(z) = \frac{\sin z}{z}$, has ------------ type of singularity at z = 0
12. Identify the poles of $\frac{3z^2 - 1}{(z^2-2iz)^3}$.

(12 x 1= 12 Marks)

PART – B

Answer any ***ten*** questions. Each question carries 4 marks.

13. Prove that $|z_1 + z_2| \leq |z_1| + |z_2|$.
14. Find the locus of the point z satisfying $|z - 1| + |z + 1| = 3$.
15. Show that $f'(z)$ does not exist at any point for $f(z) = \bar{z}$.
16. Find the harmonic conjugate of $u = \sinh x \sin y$.
17. Prove that differentiable functions are continuous.
18. Find all solutions of $e^z = 2$.
19. Find the real and imaginary parts of cos z.
20. Find the principal value of $(1 - i)^{4i}$.
21. Evaluate $\int_{|z|=2} \left(\frac{z^2-1}{z^2+1}\right) dz$.

22. Evaluate $\int_{|z|=2} \frac{z^3}{(z+1)^3} dz$.

23. Find Maclaurin's series representation for sin z.

24. If a power series $\sum_{n=0}^{\infty} a_n (z - z_0)^n$ converges at $z = z_1, z_1 \neq z_0$, then prove that it is absolutely convergent at each point in the open disk $| z - z_0 | < R_1$ where $R_1 = | z_1 - z_0|$

25. Evaluate $\int_0^{\pi} \left(\frac{\cos 2\theta}{5+4\cos\theta}\right) d\theta$.

26. Find the Cauchy Principal Value of the integral $\int_{-\infty}^{\infty} \left(\frac{x \sin x}{x^2+2x+2}\right) dx$.

(10 x 4 = 40 Marks)

PART – C

Answer any ***six*** questions. Each question carries 7 marks.

27. Prove the necessary condition for a complex function to be differentiable at a point.

28. State and prove Cauchy Integral formula.

29. Show that the function $f(z) = \sqrt{xy}$ is not analytic at the origin even though Cauchy Riemann equations are satisfied at that point.

30. Show that the derived series has the same radius of convergence as the original series.

31. State and prove Liouville's theorem.

32. Obtain Laurent series expansion of $\frac{3z+7}{(z+2)(z+3)}$ in $2 < |z| < 3$

33. Prove that a power series $S(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n$ is a continuous function at each point inside the circle of convergence.

34. Use multiplication of series to show that

$$\frac{e^z}{z(z^2+1)} = \frac{1}{z} + 1 - \frac{z}{2} - \frac{5}{6}z^2 + \cdots (0 < |z| < 1)$$

35. State and prove Cauchy Residue theorem.

(6 x 7= 42 Marks)

PART – D

Answer any ***two*** questions. Each question carries 13 marks.

36. State and prove Taylor's theorem.

37. Obtain all Laurent series expansions of $f(z) = \frac{-1}{(z-1)(z-2)}$ about z = 0.

38. a) Find the residues of $f(z) = \frac{2z^2+1}{z^3+3z^2+2z}$ at its poles.

    b) Using method of residues, find $\int_{-\infty}^{\infty} \left(\frac{2x^2-1}{x^4+5x^2+4}\right) dx$.

(2 x13= 26 Marks)

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