15U604 (Pages: 2) Name: …………..……….

Reg. No……….……….

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2018

(CUCBCSS-UG)

CC15U MAT6 B12 - NUMBER THEORY AND LINEAR ALGEBRA

Mathematics - Core Course

(2015 Admission)

Time: Three Hours Maximum: 120 Marks

PART-A

(Answer ***all*** questions. Each question carries 1 mark.)

1 *True or false:* Let V be a finite dimensional Vector Space with dimension 5. Minimal spanning set of V consists of minimum 5 elements.

2 Give an example for infinite dimensional vector space.

3 Write standard basis of $\mathbb{R}^3$ over $\mathbb{R}$.

4 Determine a linear map $f: \mathbb{R}^3 \to \mathbb{R}^3$ such that $Imf = \{(x, o, z): x, z \in \mathbb{R}\}$

5 If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is linear and $T(0,1) = (1, 1)$, then $T(0, -1) =$…………..

6 The linear Diophantine equation $ax + by = c$ has a solution if and only if…….

7 For a prime p, define $p^{\#}$?

8 Give an example for pseudo prime.

9 State the converse of Wilson's theorem.

10 $\sum_{d/100} \phi(d) =$……

11 Give an example for a non multiplicative number theoretic function.

12 $\sigma(16) =$…………….. **(12 x 1 =12 Marks)**

PART-B

(Answer any ***ten*** questions. Each question carries 4 marks.)

13. Show that the set $L = \{(x, y): \alpha x + \beta y = 0; x, y \epsilon \mathbb{R}\}$ is a subspace of the real space $\mathbb{R}^2$.
14. Find the value of m, such that the vector $(m, 7, -4)$ is a linear combination of vectors (-2, 2, 1) and (2, 1, -2).
15. Decide whether $f: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $f(x, y, z) = (x + 2, y, z)$ is linear or not?
16. If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is given by $T(a, b) = (b, 0)$, prove that ImT = Ker T.
17. Prove that $\mathbb{R}^3$, the set of all 3 – tuples of real numbers is a vector space over $\mathbb{R}$.
18. Determine the subspace of $\mathbb{R}^3$with dimension 2.
19. Find all prime numbers that divide 20!
20. If n is a square free integer, prove that $\tau(n) = 2^r$, where r is the number of prime divisors of n.
21. Show that the square of any odd integer is of the form $8k + 1$.
22. Determine all positive integer solutions of the Diophantine equation $54x + 21y = 906$.

23. Show that $3^{2n} + 24n \equiv 1(\, mod\ 32)$

24. Find the solution of the system of congruences:

$$3x + 4y \equiv 5(mod 13)$$
$$2x + 5y \equiv 7(mod 13)$$

25. Find the remainder when 18! is divided by 23.

26. Find the last two digits in the decimal representation of 3^{256}.

(10 x 4 =40 Marks)

PART-C

(Answer any ***six*** questions. Each question carries 7 marks.)

27. Let V be a vector space over a field F. If S is a subset of V that contains at least two elements, prove that S is linearly dependent if and only if at least one element of S can be expressed as a linear combination of the other elements of S.

28. If V has a finite basis B, prove that every basis of V is finite and has the same number of elements as B.

29. Let V be a finite dimensional vector space. If G is a finite spanning set of V and if I is a linearly independent subset of V such that I ⊆ G, prove that there is a basis B of V such that I $\subseteq B \subseteq G$.

30. Let V be a vector space of dimension $n \geq 1$ over a field F. Prove that V is isomorphic to the vector space F^n.

31. State and prove fundamental theorem of Arithmetic.

32. For arbitrary integers a and b, prove that $a \equiv b\ (mod\ n)$if and only if a and b leave the same nonnegative remainder when divided by n.

33. State and prove Fermat's Theorem.

34. If n is a positive integer and p a prime, prove that the exponent of the highest power of p that divides n! is $\sum_{k=1}^{\infty}[\frac{n}{p^k}]$.

35. If f is a multiplicative function and F is defined by $F(n) = \sum_{d/n} f(d)$, prove that F is also multiplicative.

(6 x 7 =42 Marks)

.PART-D

(Answer any ***two*** questions. Each question carries 13 marks.)

36. State and prove dimension Theorem.

37. State and prove Euclidean Algorithm for finding the g.c.d.

38. (a) Let f and F be number – theoretic functions such that $F(n) = \sum_{d/n} f(d)$.

    Prove that $\sum_{n=1}^{N} F(n) = \sum_{k=1}^{N} f(k) \left[\frac{N}{k}\right]$, for any positive integer N.

    (b) Verify the result $\sum_{n=1}^{N} \sigma(n) = \sum_{n=1}^{N} n \left[\frac{N}{n}\right]$ for N = 6.

(2 x 13 =26 Marks)

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