16U602 (Pages: 2) Name: ……………………

Reg. No……………………

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2019

(Regular/Supplementary/Improvement)

(CUCBCSS-UG)

CC15U MAT6 B10 - COMPLEX ANALYSIS

Mathematics - Core Course

(2015 Admission onwards)

Time: Three Hours Maximum: 120 Marks

Part-A

Answer ***all*** questions. Each question carries 1 mark.

1. Write the Cauchy Reimann equations in polar form.
2. Verify the function u = xy is harmonic or not.
3. Show that $e^{(1+i)\pi} = -e^{\pi}$
4. Define Jordan arc.
5. Find $\int_{|z|=1}\left(\frac{z^2}{z-2}\right)dz$
6. State Morera's Theorem
7. Find the power series representation of $\frac{1}{1-z}$ in negative powers of z
8. If R is the radius of convergence of $\sum_{n=0}^{\infty} a_n z^n$ then find the radius of convergence of $\sum_{n=0}^{\infty} a_n z^{2n}$
9. If $z_n = (2+i) + \left(\frac{2i-1}{n}\right)$, then find $\lim_{n\to\infty} z_n$
10. Which type of the singularity does the function $f(z) = \frac{1}{sin\frac{\pi}{z}}$ has at z = 0
11. Find the residue of $f(z) = e^{\frac{2}{z}}$ at z = 0
12. Define Cauchy principal value of the integral $\int_{-\infty}^{\infty} f(x)dx$

(12 x 1 = 12 Marks)

Part-B

Answer any t***en*** questions. Each question carries 4 marks.

13. Show that f(z) = z Re(z) is nowhere analytic.
14. If f(z) = u + iv is analytic in a domain D then show that u and v are harmonic in D.
15. Find the real and imaginary parts of sinhz.
16. Define a branch cut and write the branch cut for the complex valued function log z
17. Evaluate $\int_{|z|=2} \bar{z}dz$
18. State and prove principle of deformation of path.
19. Find $\oint_C \frac{1}{z-i}dz$ where C is the boundary of the triangle with vertices -1, 1, 2i in clockwise sense using Cauchy integral formula.
20. Find the Taylor series expansion for cosz about z = $\frac{\pi}{2}$

21. State Laurent's Theorem.
22. If $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n$ = y, then show that $\lim_{n\to\infty} z_n = z$ where $z_n = x_n + i\, y_n$ and $z = x + iy$
23. Find the residues at singular points of the function $\frac{z^3+2z}{(z-i)^3}$
24. Using Cauchy's residue theorem evaluate $\int_C \frac{2z^2+2}{z^2-1}dz$ where C is the circle | z-1 |= 1 in counter clockwise sense.
25. Define isolated singular point at infinity and residue at infinity.
26. Evaluate the integral $\int_0^\infty \frac{1}{x^2+1}$ dx using residues.

(10 x 4 = 40 Marks)

Part-C

Answer any ***six*** questions. Each question carries 7 marks.

27. Derive the Cauchy Reimann equations of analytic functions.
28. If $u - v = (x-y)(x^2+4xy+y^2)$ and $f(z) = u + iv$ is analytic function of $z = x + iy$ then find f(z) in terms of z
29. Find all values of
    a) $\sin^{-1}$(-i)
    b) $\tan^{-1}$(2i)
30. Let C be the arc of the circle |z| = 2 from z= 2 to z = 2i that lies in the first quadrant. Show that $|\int_C \frac{1}{z^2-1}dz| \leq \frac{\pi}{3}$ without evaluating the integral.
31. State and prove Cauchy's in equality.
32. Find all Laurent series of $\frac{1}{z^3-z^4}$ with centre 0
33. a) Define Circle of convergence and radius of convergence of the power series.
    b) Find the circle of convergence and radius of convergence of the series $\sum_{n=0}^{\infty} \frac{n!}{n^n} z^n$
34. State and prove Cauchy's Residue Theorem.
35. Show that $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$ $a > b > 0$

(6 x 7 = 42 Marks)

Part-D

Answer any ***two*** questions. Each question carries 13 marks.

36. If f(z) is a regular function of z, then show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4\,|f'(z)|^2$
37. State and prove Cauchy's Integral formula.
38. a) Show that $\int_{-\infty}^{\infty} \left(\frac{2x^2-1}{x^4+5x^2+4}\right) dx = \frac{\pi}{2}$
    b) If a > 0, b > 0 then show that $\int_0^\infty \left(\frac{\cos ax}{x^2+b^2}\right) dx = \frac{\pi}{2b} e^{-ab}$

(2 x 13 = 26 Marks)

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