16U604 (Pages: 2) Name: ……………………….

Reg. No……………………

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2019

(Regular/Supplementary/Improvement)

(CUCBCSS-UG)

CC15U MAT6 B12 - NUMBER THEORY AND LINEAR ALGEBRA

Mathematics - Core Course

(2015 Admission onwards)

Time: Three Hours Maximum: 120 Marks

Section A

Answer ***all*** questions. Each question carries 1 mark.

1. Find the gcd of 7469 and 3387
2. What is Euclid's Lemma?
3. Show that the product of any two integers of the form $3k+1$ is also of the same form.
4. Explain the Sieve of Eratosthenes.
5. Show by an example that $a^2 \equiv b^2 \pmod{n}$ need not imply $a \equiv b \pmod{n}$
6. Give the binary representation of the number 372 which is in the decimal representation.
7. Find the number and sum of divisors of 360
8. Is $\{(x, y, z, t) / x = y, z = t\}$ where x, y, z, t are real numbers is a subspace of $\mathbb{R}^4$
9. Whether $S = \{(1,1), (1,-1)\}$ is a basis of $\mathbb{R}^2$. Explain why?
10. Is $T: \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(x, y) = (x-y, x, y)$ is linear.
11. If $f: U \to V$ is linear and if f is injective, show that Ker $f = \{0_v\}$
12. Show that if U and V are vector spaces of the same dimension, then U and V are isomorphic.

(12× 1 = 12 Marks)

Section B

Answer any ***ten*** questions. Each question carries 4 marks.

13. Prove that $(n(n+1)(2n+1)) / 6$ is an integer for $n \geq 1$
14. Use Euclidean Algorithm to find the integers x and y, such that gcd $(24,138)=24x + 138y$
15. Determine all solutions of the Diophantine equation $172x + 20y = 1000$
16. If p is a prime and p / ab, then prove that ${}^{p}/_{a}$ or ${}^{p}/_{b}$
17. Prove that the number of primes is infinite.

18. If N = $a_m 10^m + a_{m-1}10^{m-1} + \cdots + a_1 10 + a_0$ be the decimal expansion of the positive integer N , $0 \le a_k < 10$ and let S = $a_0 + a_1 + a_2 + \ldots\ldots a_{m-1} + a_m$, then prove that $\frac{9}{N}$ if and only if $\frac{9}{S}$
19. Show that $n^7 - n$ is divisible by 42
20. Find the remainder when 15! is divided by 17
21. Find the smallest number with 20 divisors.
22. If p is a prime number and $k > 0$, then prove that $\varphi(p^k) = p^k\left(1 - \frac{1}{p}\right)$
23. Show that if S is a nonempty subset of a vector space V, then span S is a subspace of V
24. If V has a finite basis then prove that all linearly independent subsets of V are finite.
25. Determine the subspaces of $\mathbb{R}^2$ with dimensions 0, 1 and 2
26. Show that the map $T:\mathbb{R}^3 \to \mathbb{R}_2[x]$ defined by T (a, b, c)= $a + bcx + x^2$ is not linear.

($10 \times 4 = 40$ Marks)

Section C

Answer any ***six*** questions. Each question carries 7 marks.

27. Is the set of symmetric $n \times n$ matrices is a subspace of the set of all $n \times n$ matrices over $\mathbb{R}$?
28. Solve $x \equiv 1 (\text{mod } 3), x \equiv 2 (\text{mod } 5), x \equiv 3 (\text{mod } 7)$
29. State and prove Wilsons Theorem.
30. Prove that gcd $(a + b,\ a^2 + b^2)$ =1 or 2
31. State and prove Division Algorithm.
32. Using congruences solve the Diophantine equation $15x + 21y = 39$
33. If $\{u, v, w\}$ is linearly independent, show that $\{u - v,\ v - w,\ w - u\}$ is linearly independent.
34. Let $T: \mathbb{R}^4 \to \mathbb{R}^3$ be a linear map defined by $T(x,\ y,\ z,\ t) = (x - y,\ 0,\ z - t)$, then find null space, image, nullity and rank of T.
35. Let $f: U \to V$ be a linear map. Prove that if X is a subspace of U then $f^{\rightarrow}(X)$ is a subspace of V and if Y is a subspace of V then $f^{\leftarrow}(Y)$ is a subspace of U.

($6 \times 7 = 42$ Marks)

Section D

Answer any ***two*** questions. Each question carries 13 marks.

36. State and prove Chinese Remainder Theorem.
37. State and prove Unique Factorisation Theorem.
38. State and prove Dimension Theorem.

($2 \times 13 = 26$ Marks)

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