20U108A (Pages: 2) Name: ....

Reg.No: ....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2020

(CBCSS - UG)

CC20U MTS1 B01 - BASIC LOGIC AND NUMBER THEORY

(Mathematics - Core Course)

(2020 Admission - Regular)

Time : 2.5 Hours Maximum : 80 Marks

Credit: 4

Part A (Short answer questions)

Answer ***all*** questions. Each question carries 2 marks.

1. Evaluate the boolean expression $\sim [(a > b) \vee (b \leq d)]$ where $a = 2, b = 3, c = 5$ and $d = 7$
2. Determine whether $p \vee (\sim p)$ is a tautology.
3. Verify $\sim (\sim p \vee \sim q) \equiv p \wedge q$.
4. Negate each quantified propositions
   a) Every computer is a 16-bit machine. b) All chalkboards are black.
5. Write a short note on:
   a) Simplification law b) Hypothetical syllogism.
6. Define recursively the number sequence $0, 3, 9, 21, 45, \ldots\ldots..$
7. Find the number of positive integers is less than or equal to 3076 and not divisible by 17.
8. State the prime number theorem.
9. Find the five consecutive composite numbers less than 100.
10. Express (28, 12) as a linear combination of 28 and 12
11. Can every integer greater than or equal to 2 can be decomposed into primes?
12. State Dirichlet's Theorem
13. Prove the symmetric property of congruene modulo m.
14. Let p be a prime and a any integer such that p does not divide a. Then show that a^{p-2} is an inverse of a modulo p.
15. Define Euler's phi function and compute $\phi(18)$. **(Ceiling: 25 Marks)**

Part B (Paragraph questions)

Answer ***all*** questions. Each question carries 5 marks.

16. Let S be the subset of $\mathbb{N}$ that preserves the two properties:

    (i) The number $1 \in S$.

    (ii) For every $k \in \mathbb{N}$, if $k \in S$, then $k+1 \in S$.

    Then prove that $S = \mathbb{N}$.

17. Using the Euclidean Algorithm, Find the gcd of 1024, 1000

18. Find the positive factors of 60.

19. Prove that if $m_1, m_2, \dots . m_k$ and a be positive integers such that $m_i | a$ for $1 \leq i \leq k$, then $[m_i, m_2 \dots . m_k] | a$.

20. Solve the congruence $12x \equiv 18 (mod 15)$.

21. If p is a prime, then show that $(p-1)! \equiv -1$ (mod p)

22. Let p be a prime and a any integer such that p does not divide a. Then show that $a^{p-1} \equiv 1 (mod p)$.

23. Solve the linear congruence $7x \equiv 8$ (mod 10)

(Ceiling: 35 Marks)

Part C (Essay questions)

Answer any ***two*** questions. Each question carries 10 marks.

24. (i) Explain

    a) Proof of contrapositive b) Direct proof c) Proof by cases

    d) Constructive existence proof e) Counter example method

    (ii) Prove by contradiction;

    There is no largest prime number; that is, there are infinitely many prime numbers.

25. If a cock is worth five coins, a hen three coins and three chicks together one coin. How many cocks, hens and chicks, totaling 100, can be bought for 100 coins?

26. a) Show that $f(5) = 2^{2^5} + 1$ is divisible by 641.

    b) Find the last digit in the decimal value of $1997^{1998^{1999}}$

27. a) Using congruences solve $x + y + z = 100$

    $$5x + 3y + \frac{z}{3} = 100.$$

    b) Using inverses, find the incongruent solution of $48x \equiv 39$ (mod 17).

(2 × 10 = 20 Marks)

****